Seasonal hydropower scheduling using linear decision rules

Knut Hjertvik
Simen Braaten

NTNU
powel

Agenda

- LDR approximation vs scenarios
- Modelling considerations
- Description of benchmarking
- How does LDR perform?
- Analysis of results

- ~~Uncertainty modelling~~
- ~~Mathematical transformations~~
- ~~How head was included~~
- ~~How water values was calculated~~

Problem and solution method

- Seasonal Hydro Power Planning Problem
  - Weekly resolution
  - Uncertainty in price and inflow
  - Plan power generation, pumping, bypass
  - For input to short-term models
- Linear Decision Rules
  - Affine reaction functions

$$
\begin{array}{lll}
\max & T^{H} \sum\limits_{t \in \mathcal{T}} \beta_t \pi_t \sum\limits_{r \in \mathcal{R}} \sum\limits_{d \in \mathbb{D}} E_{rt}^{d} x_{rt}^{d} & \\
\text{s.t.} & m_{r0} = M_r^0 & r \in \mathcal{R} \\
 & m_{rt} = m_{r(t\text{-}1)} + F_{rt} + T^{S} \sum\limits_{d \in \mathbb{D}} \left(\sum\limits_{\rho \in \mathbb{C}_r^d} x_{\rho t}^{d} - x_{rt}^{d} \right) & r \in \mathcal{R}, t \in \mathcal{T} \\
 & \underline{M}_{rt} \leq m_{rt} \leq \overline{M}_{rt} & r \in \mathcal{R}, t \in \mathcal{T} \\
 & L_{rt}^{q} \leq E_{rt}^{q} x_{rt}^{q} \leq U_{rt}^{q} & r \in \mathcal{R}, t \in \mathcal{T} \\
 & L_{rt}^{p} \leq E_{rt}^{p} x_{rt}^{p} \leq U_{rt}^{p} & r \in \mathcal{R}, t \in \mathcal{T} \\
 & \underline{D}_{rt} \leq x_{rt}^{q} + C_{rt}^{B} \leq \overline{D}_{rt} & r \in \mathcal{R}, t \in \mathcal{T} \\
 & x_{rt}^{s} \geq 0 & r \in \mathcal{R}, t \in \mathcal{T}
\end{array}
$$

Problem and solution method

- Seasonal Hydro Power Planning Problem
  - Weekly resolution
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- Linear Decision Rules
  - Affine reaction functions
  - Policy: Intercept + slope

Intercept

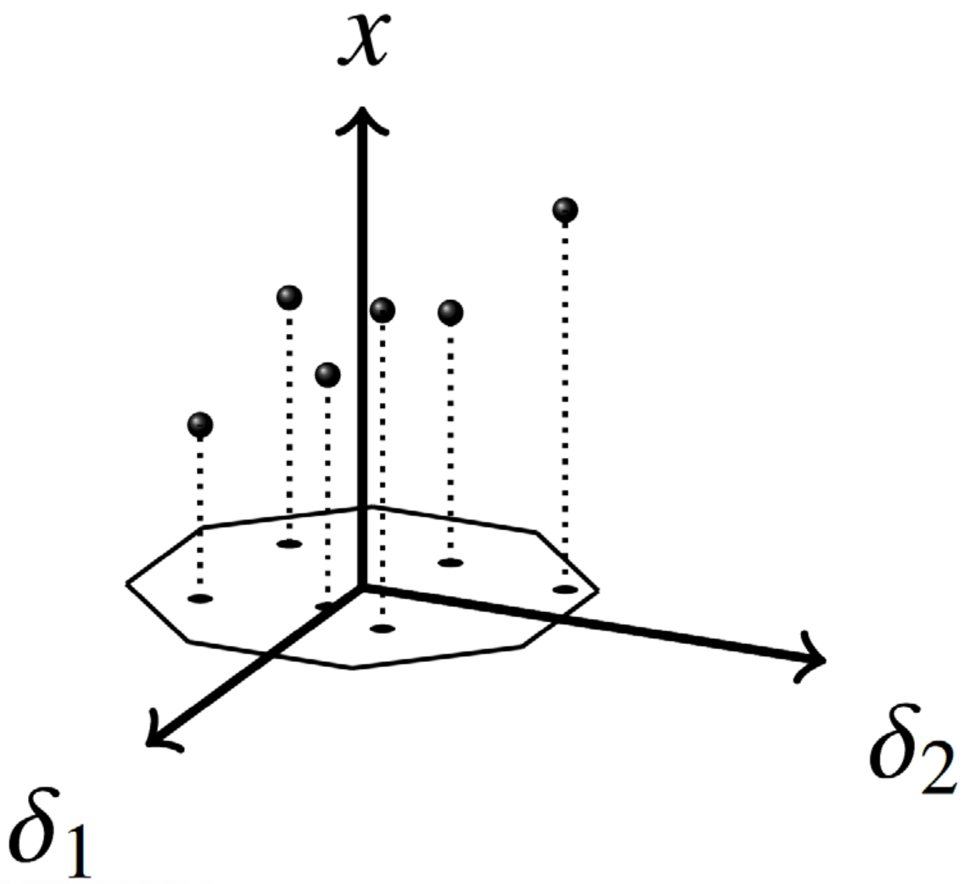
Uncertainty parameter

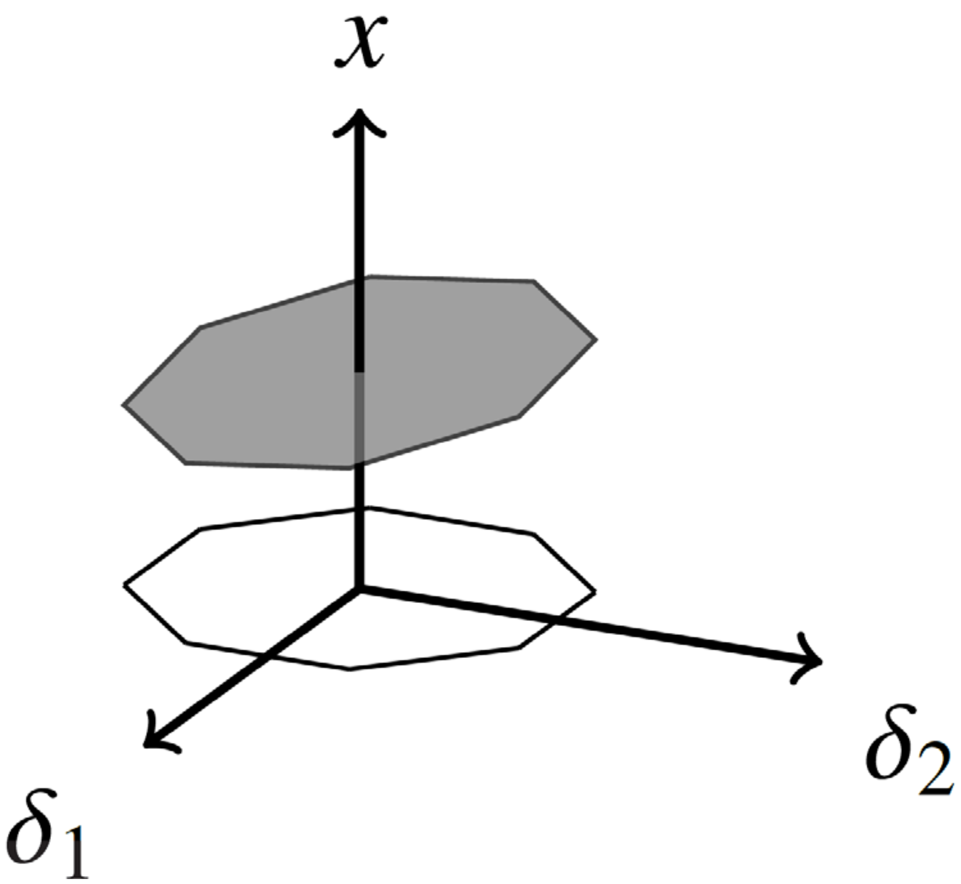
$$\mathbf{x}(\delta) = \hat{\mathbf{x}} + K^x \delta$$

Decision rule

Slope

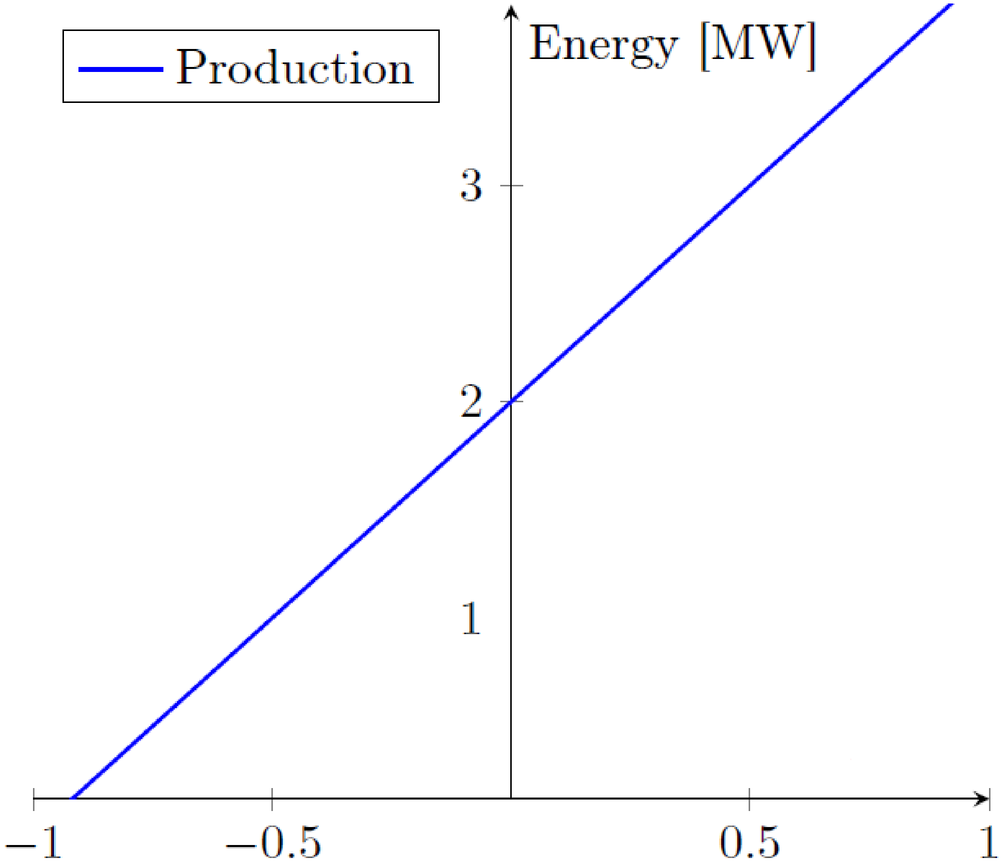
LDR approximation





Modelling considerations 1

Zero-mean uncertainty



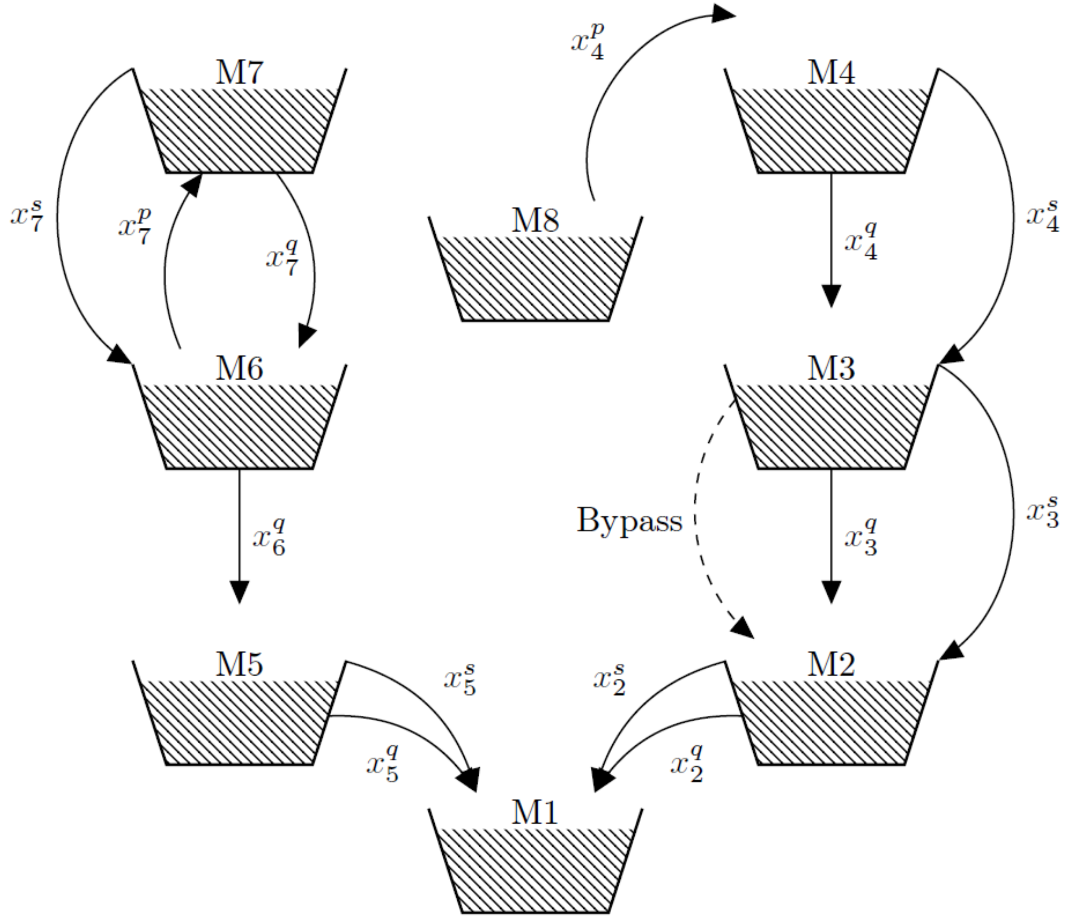
Modelling considerations 2

Memory sets

- On what time periods can the decisions react?
- Solution time vs. optimality gap
- Smart memory sets

Benchmarking

Description of test case



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Benchmarking

Description of testing

- **Solution methods**
  - Linear Decision Rules (LDR)
    - Different memory sets
  - Rolling Intrinsic (RI)
  - Dual Upper Bound (DUB)

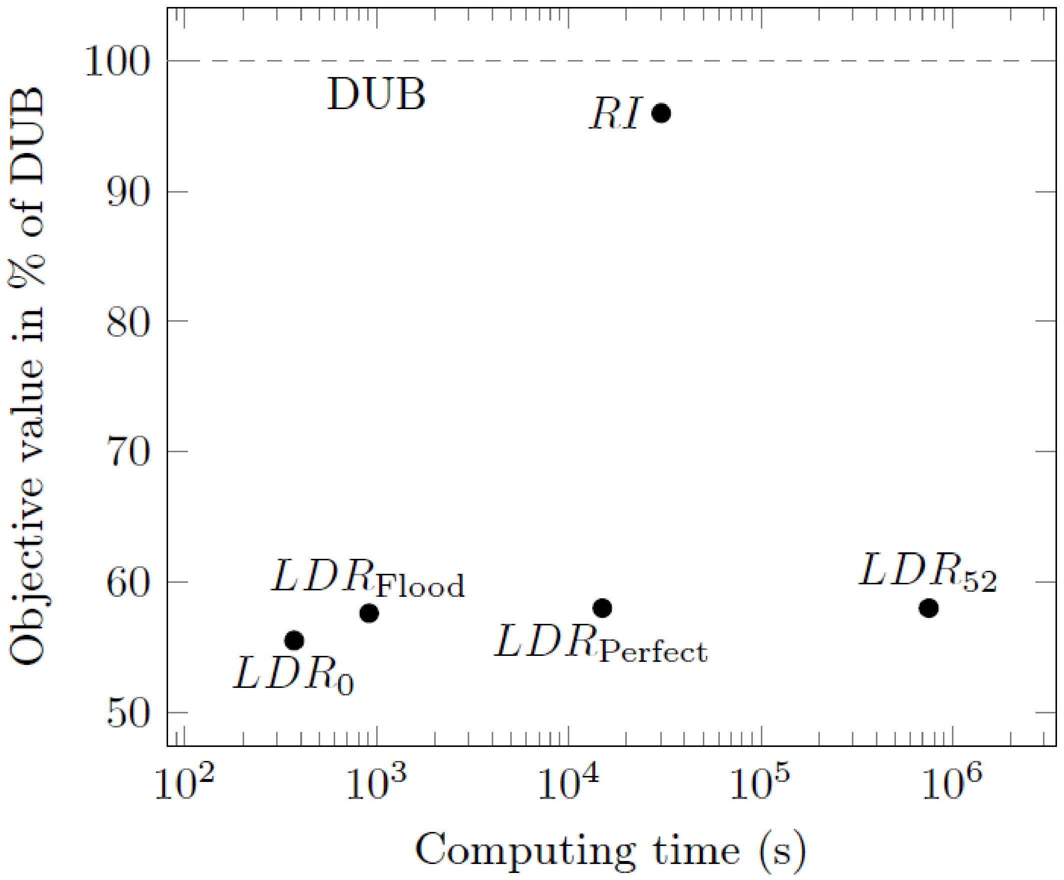
- Simulated policies on 3000 scenarios

Results

Analysis

- Computing time
  - Different memory sets
  - Compared to RI
- Solution quality
  - Optimality gap
- Why?

$$\mathbf{x}(\delta) = \hat{\mathbf{x}} + K^x \delta$$



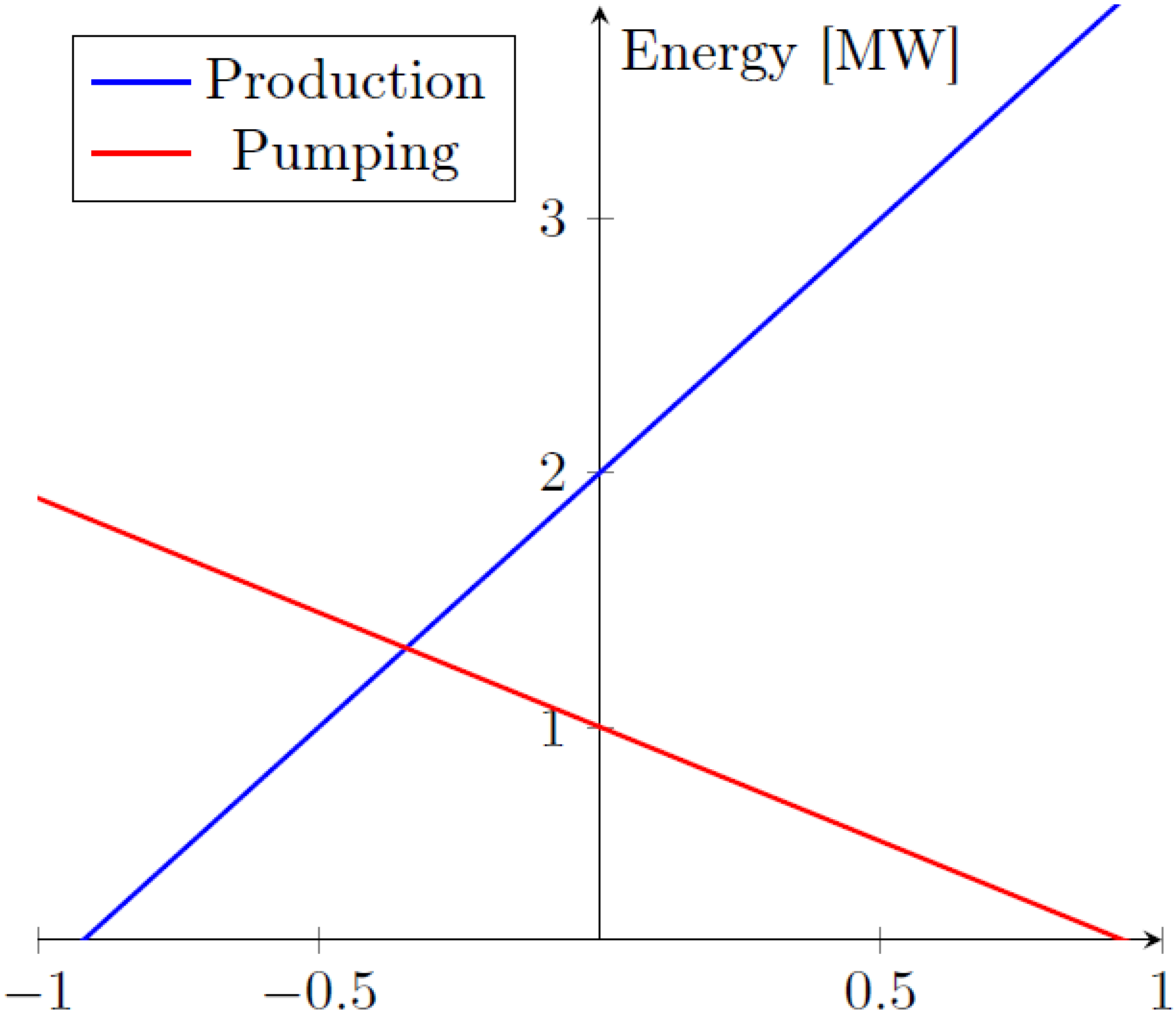
Results

Analysis

- ***Linear*** Decision Rules

Results

Analysis



Results

Analysis

- *Linear* Decision Rules
- Further research
  - Piecewise Linear Decision Rules
  - Historically based memory sets

Conclusion

- Know the strengths of LDR
  - Swift on smaller memory sets
  - Potential in piece-wise LDR
- Be aware of the limitations of LDR
  - Loss of optimality
  - Linearity of model

Thank you!

Our master thesis:
https://daim.idi.ntnu.no/masteroppgave?id=13112

Questions or other inquiries? Send me an email at
Knut.Hjertvik@powel.com

Knut Hjertvik
+47 908 44 997

Attachements

Heat maps

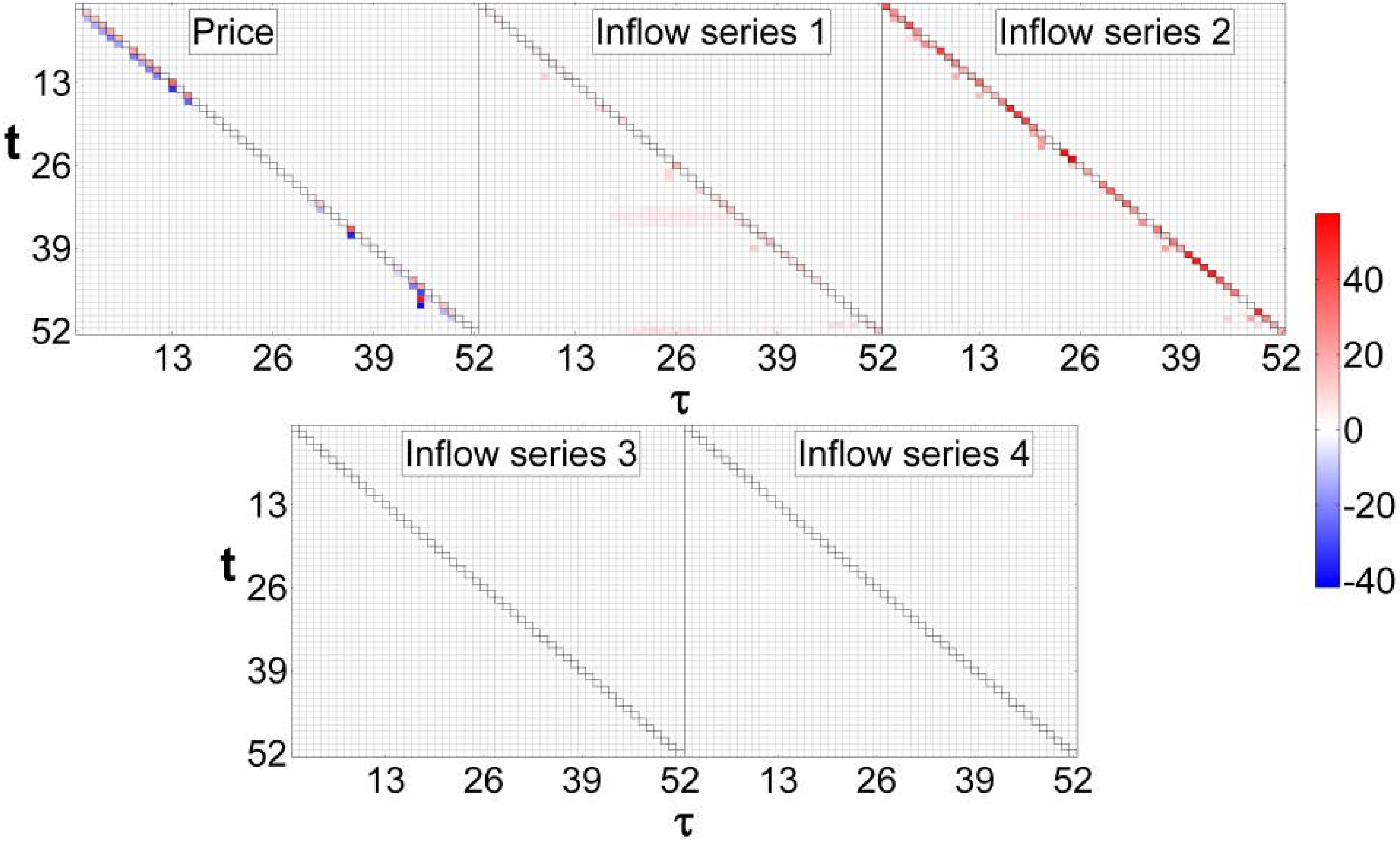
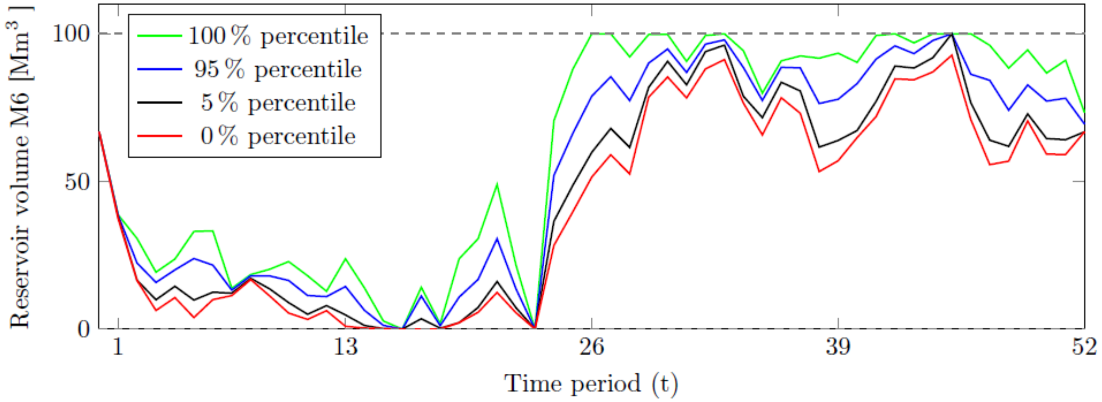
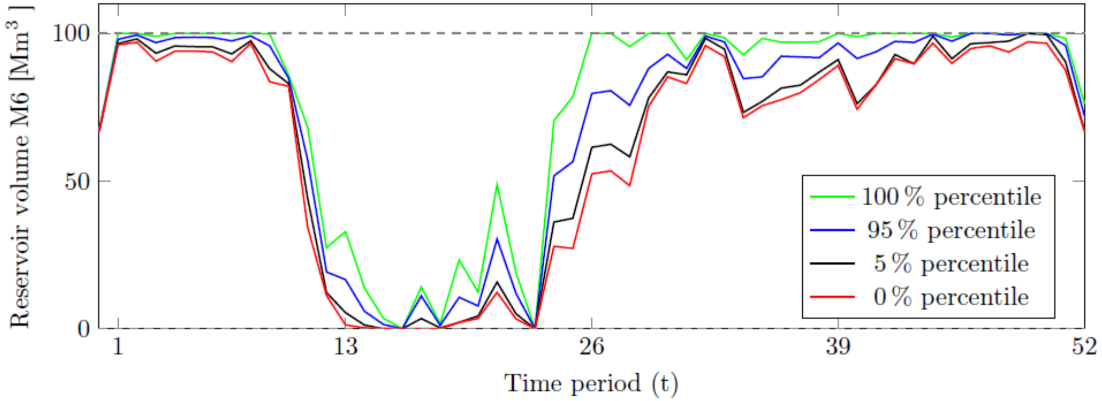


Figure 4: Optimal slope matrix for discharge from reservoir M6

Inclusion of head corrections



(a) Reservoir trajectories M6 prior to head corrections

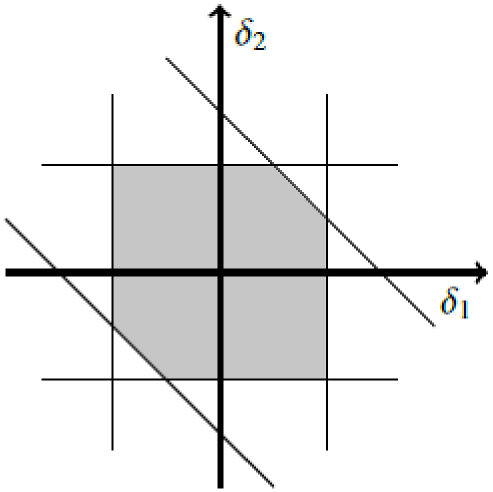


(b) Reservoir trajectories M6 after head corrections

Inclusion of uncertainty

- Uncertainty polytope
  - Box Uncertainty
  - Budgeted Uncertainty sets

$$
\begin{array}{lll}
-1 \leq & \delta_{ut} \leq 1 & u \in \mathbb{U},\ t \in \mathcal{T} \\
-\Gamma_i^L \leq & \sum_{t \in \mathcal{T}} \delta_{ut} \leq \Gamma_i^U & u \in \mathbb{U},
\end{array}
$$



Mathematical transformation

$$
\begin{array}{lllr}
\max & T^{H}\sum_{t\in\mathcal{T}}\beta_t\pi_t\sum_{r\in\mathcal{R}}\sum_{d\in\mathbb{D}}E_{rt}^{d}x_{rt}^{d} & & (1)\\
\text{s.t.} & m_{r0} = M_r^{0} & r\in\mathcal{R} & (2)\\
 & m_{rt} = m_{r(t\text{-}1)} + F_{rt} + T^{S}\sum_{d\in\mathbb{D}}\left(\sum_{\rho\in\mathbb{C}_r^{d}}x_{\rho t}^{d} - x_{rt}^{d}\right) & r\in\mathcal{R}, t\in\mathcal{T} & (3)\\
 & \underline{M}_{rt} \le m_{rt} \le \overline{M}_{rt} & r\in\mathcal{R}, t\in\mathcal{T} & (4)\\
 & L_{rt}^{q} \le E_{rt}^{q}x_{rt}^{q} \le U_{rt}^{q} & r\in\mathcal{R}, t\in\mathcal{T} & (5)\\
 & L_{rt}^{p} \le E_{rt}^{p}x_{rt}^{p} \le U_{rt}^{p} & r\in\mathcal{R}, t\in\mathcal{T} & (6)\\
 & \underline{D}_{rt} \le x_{rt}^{q} + C_{rt}^{B} \le \overline{D}_{rt} & r\in\mathcal{R}, t\in\mathcal{T} & (7)\\
 & x_{rt}^{s} \ge 0 & r\in\mathcal{R}, t\in\mathcal{T} & (8)
\end{array}
$$

$$\hat{x}^q_{rt} + \sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{M}^u_t} K^q_{rt\tau u}\delta_{u\tau} \le \frac{U^q_{rt}}{E^q_{rt}}, \quad r\in\mathcal{R}, t\in\mathcal{T}, \delta\in\mathcal{U}. \tag{14}$$

$$\max_{\delta}\left\{\begin{array}{l} \displaystyle\sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{M}^u_t} K^q_{rt\tau\upsilon}\delta_{u\tau} \\ \displaystyle s.t. \sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{T}} H_{ju\tau}\delta_{u\tau} \le h_j, \quad \forall j \quad : \mu^{Uq}_{rtj} \end{array}\right\} \le \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt} \qquad r\in\mathcal{R}, t\in\mathcal{T}$$

$$\min_{\mu^{Uq}}\left\{\begin{array}{lll} \displaystyle\sum_{j} h_j\mu^{Uq}_{rtj} & & \\ \displaystyle s.t. \sum_{j} H_{ju\tau}\mu^{Uq}_{rtj} = K^q_{rt\tau\upsilon}, & u\in\mathbb{U}, \tau\in\mathcal{M}^u_t & : \delta_{u\tau} \\ \displaystyle\sum_{j} H_{ju\tau}\mu^{Uq}_{rtj} = 0, & u\in\mathbb{U}, \tau\notin\mathcal{M}^u_t & : \delta_{u\tau} \\ \mu^{Uq}_{rtj} \ge 0, \quad \forall j & & \end{array}\right\} \le \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt} \qquad r\in\mathcal{R}, t\in\mathcal{T}.$$

$$\begin{aligned}
\sum_j h_j \mu_{rtj}^{Uq} &\leq \frac{U_{rt}^q}{E_{rt}^q} - \hat{x}_{rt}^q, && r \in \mathcal{R}, t \in \mathcal{T} \\
\sum_j H_{ju\tau} \mu_{rtj}^{Uq} &= K_{rt\tau\upsilon}^q, && r \in \mathcal{R}, t \in \mathcal{T}, u \in \mathbb{U}, \tau \in \mathcal{M}_t^u \\
\sum_j H_{ju\tau} \mu_{rtj}^{Uq} &= 0, && r \in \mathcal{R}, t \in \mathcal{T}, u \in \mathbb{U}, \tau \notin \mathcal{M}_t^u \\
\mu_{rtj}^{Uq} &\geq 0, && r \in \mathcal{R}, t \in \mathcal{T}, j.
\end{aligned}$$