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# A hybrid POD approach for parametrised turbulent flow problems

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# Motivation

- Optimization of energy production of wind farm deployment.
  - ◆ Relative positioning and loading of wind-turbine array.

- Adapting to an increasingly demanding environment.
  - ◆ Multiple queries.
  - ◆ Real-time simulation.

- Nested ROM:
  - ◆ POD-Galerkin reduced model for the efficient assessment of wind-turbine flow, given environmental characteristics.
  - ◆ Compression of wind data as input of methodology (output of external ROM).
  - ◆ Model Reduction of flow around a wind-turbine.

# Hybrid POD-Galerkin method

## Methodology

### System of equations:

$$\mathbf{u}^* \cdot (\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) - \nabla \cdot (\nu_{\text{eff}} \nabla^s \mathbf{u}) + \nabla p) = \mathcal{R}_u$$
$$p^* \nabla \cdot \mathbf{u} = \mathcal{R}_p$$

### Eddy viscosity

POD:

- Reduced basis
- Parametric dependence matrix

Online:

- Interpolation of parametric coefficients

### Assumption of separability

$$\mathbf{u}^R = \sum_{i=1}^{i=N_u} \alpha^i(\mu) \phi^i(\mathbf{x})$$

$$p^R = \sum_{i=1}^{i=N_p} \beta^i(\mu) \chi^i(\mathbf{x})$$

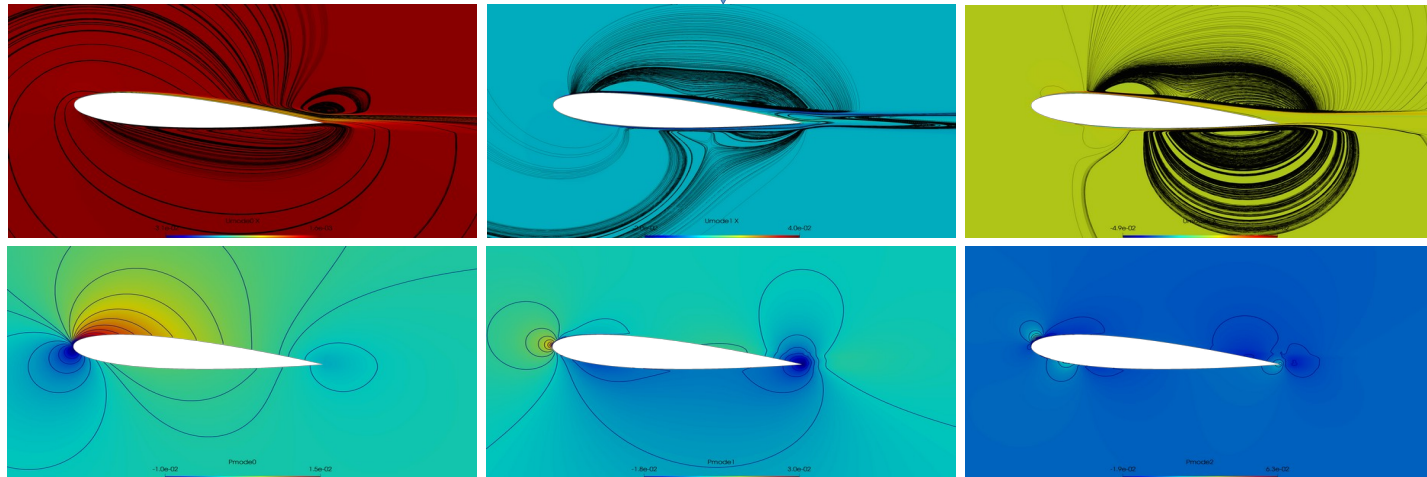
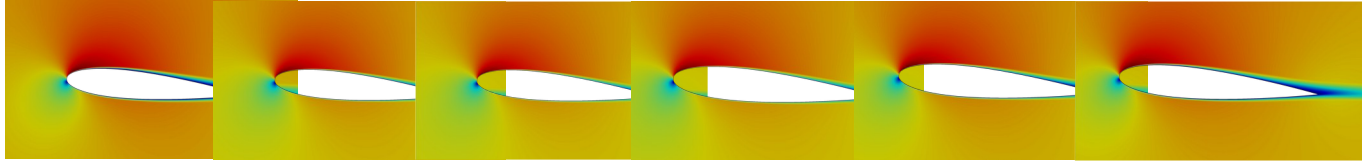
$$\nu_{\text{eff}}^R = \sum_{i=1}^{i=N_t} \gamma^i(\mu) \xi^i(\mathbf{x})$$

# Flow around a NACA 0015 airfoil

Offline phase

**Sampling**

- Angle of attack:  $10^\circ$
- Reynolds number range:  $[32 \cdot 10^3, 32 \cdot 10^4]$



**Velocity modes**

**Pressure modes**

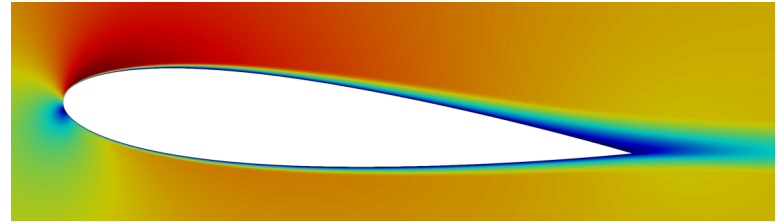
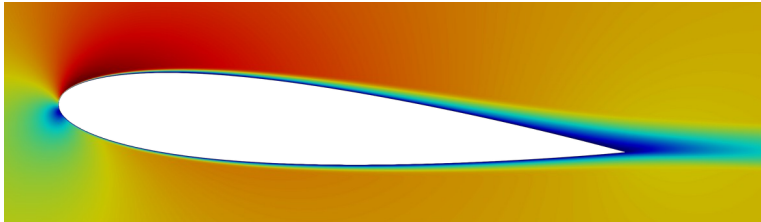
# Flow around a NACA 0015 airfoil

Online phase

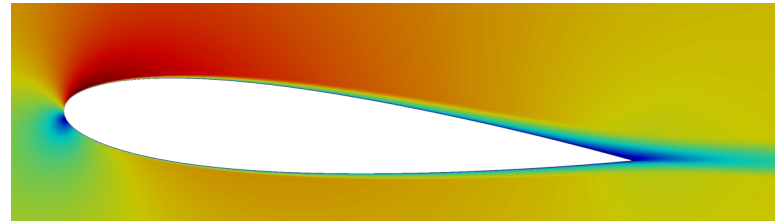
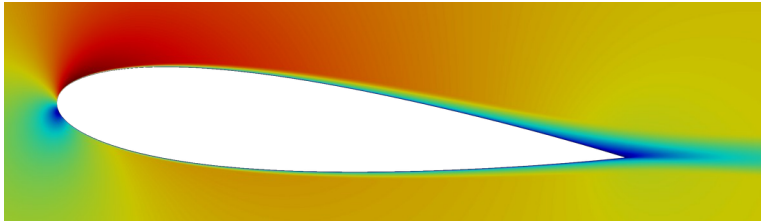
FOM

ROM

$Re = 48 \cdot 10^3$



$Re = 272 \cdot 10^3$



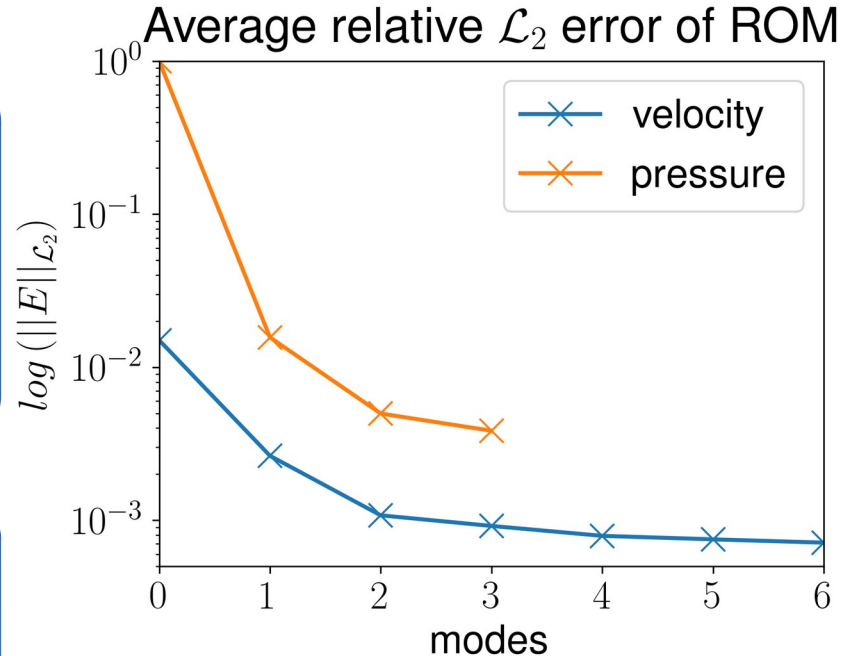
Speed-up of  $10^4$

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## Summary

- A POD-Galerkin approach was developed based on OpenFOAM's FVM solvers.
- Focus on high Reynolds numbers.
- Online phase:
  - Pressure Poisson Equation.
  - Eddy viscosity parametric dependence is computed using POD-i.

- Next steps:
  - Geometric parametrisation.
  - Further increase of Reynolds number.
  - Transient problems.



- E.F., H. v B., T.K., A.R., Fast divergence-conforming reduced basis methods for steady Navier-Stokes flow. *Computer Methods in Applied Mechanics and Engineering*, 2019
- A.Q., G.R. (editors), *Reduced Order Methods for Modeling and Computational Reduction*, Springer International Publishing, 2014