Optimal Experimental Design for Estimating Low-Frequency Hydrodynamic Loads

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Parameter Estimation

This work presents a robust approach for designing cyber-physical experiments for estimating the hydrodynamic properties of a floating structure. The work is an extension of the work presented by Sauder (2021). Without loss of generality, the motions of the system are assumed to be decoupled resulting in a set of scalar linear equations that take the following form:

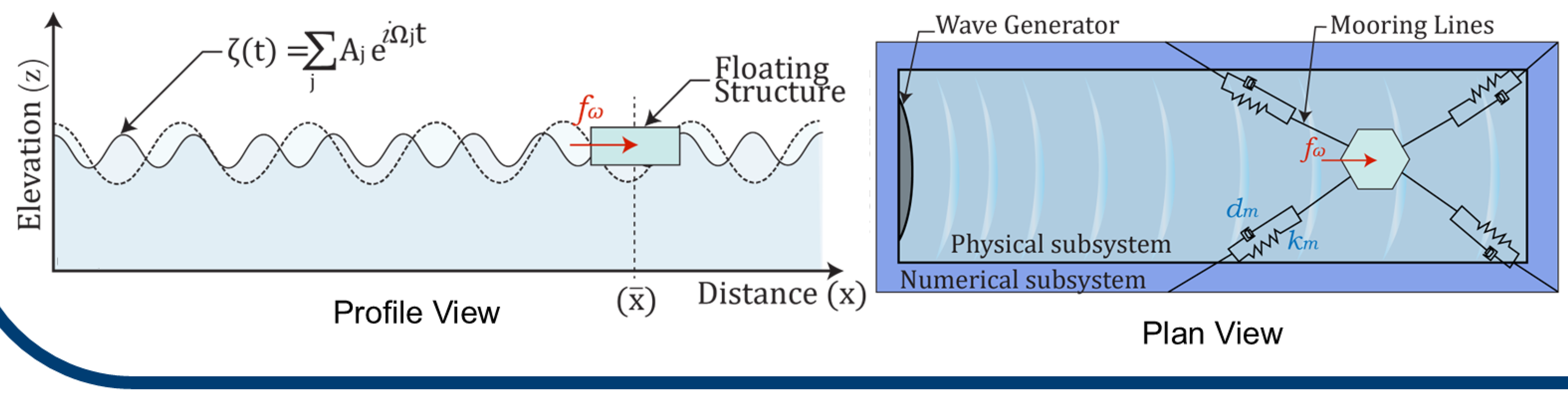
To be identified · Controlled · Measured

$$(m+\hat{a})\ddot{\eta}(t) + (\hat{d}_h + d_m)\dot{\eta}(t) + k_m \eta(t) = f_\omega(t) \qquad (1)$$

where,

- $m, \hat{a}$ – mass (structural + hydrodynamic)
- $d_m, \hat{d}_h$ – damping (mooring + hydrodynamic)
- k_m – stiffness (mooring)
- η – structural response
- f_ω – low frequency hydrodynamic force

The problem is schematically illustrated in the figure below.



The first step in the experimental design is to estimate the low frequency added mass and damping parameters from a set of experiments $i = \{1, \ldots, N\}$ with identical wave elevation profiles but different sets of mooring stiffness and damping values (k_m, d_m):

$$(m+\hat{a})\ddot{\eta}^{(i)}(t) + \left(\hat{d}_h + d_m^{(i)}\right)\dot{\eta}^{(i)}(t) + k_m^{(i)}\eta^{(i)}(t) = f_\omega^{(i)}(t) \qquad (2)$$

The key idea behind this approach is the invariance of the wave excitation $f_\omega(t)$ between experiments. Whereas the quantities $a\ddot{\eta}$ and $d_h\dot{\eta}$ are response-dependent and will vary between experiments. This means that for the correct value of $\hat{a}$ and $\hat{d}_h$ the low-frequency force time-history $f_\omega^{(i)}(t)$ will be identical between experiments. Hence, the added mass and damping can be found by minimizing a cost function proportional to the variance of the force time-history:

$$Q(a, d_h; \mathbf{k}_m, \mathbf{d}_m) = \int_0^T \mathrm{Var}_i\left[f_\omega^{(i)}(t)\right] dt \qquad (3)$$

The minimum of this function yields the correct added mass and damping:

$$\hat{a}, \hat{d}_h = \arg\min_{a, d_h} \int_0^T \mathrm{Var}_i\left[f_\omega^{(i)}(t)\right] dt \qquad (4)$$

Experimental Design

The accuracy of the method described in the block above depends, to a large extent, on the shape of the cost function defined in (3). To facilitate faster and more accurate convergence to the global minimum, we would like to design an optimal cost function, $Q(a, d_h; \hat{\boldsymbol{k}}_m, \hat{\boldsymbol{d}}_m)$. We start with the Fisher information matrix which is defined as the hessian of the logarithm of the cost function (3) with respect to the unknown parameters:

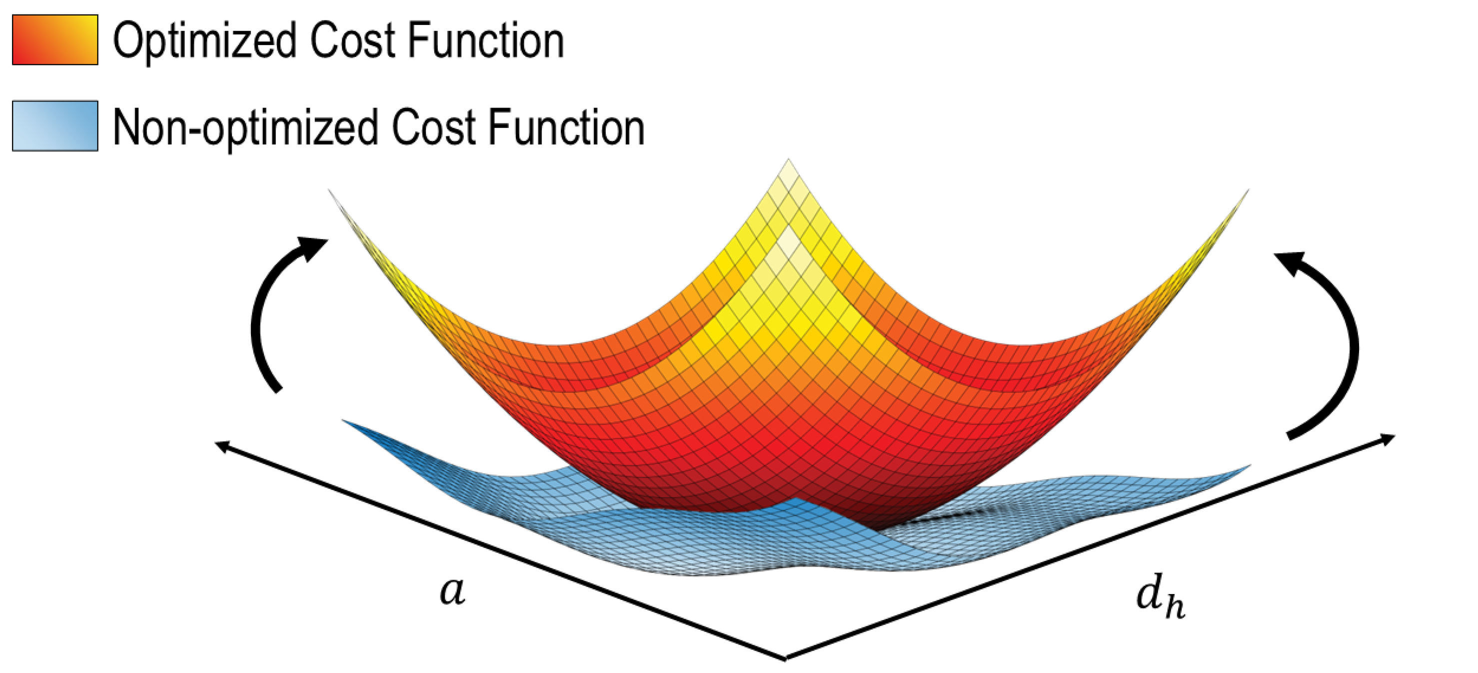
$$\mathcal{I}(a, d_h; \mathbf{k}_m, \mathbf{d}_m) = \begin{bmatrix} \dfrac{\partial^2 \log(Q)}{\partial a^2} & \dfrac{\partial^2 \log(Q)}{\partial d_h \partial a} \\ \dfrac{\partial^2 \log(Q)}{\partial a \partial d_h} & \dfrac{\partial^2 \log(Q)}{\partial d_h^2} \end{bmatrix}_{a,\, d_h} \qquad (5)$$

Note that in (5), for clarity, the dependence on the parameters is omitted. Since the true parameters $\hat{a}, \hat{d}_h$ are unknown, we resort to optimizing the expected value of (5). This is achieved using a Monte Carlo approach sampled from a uniform distribution of a, d_h with a predefined range somewhere within which the true parameters lie. Lastly, we seek D-optimality which maximizes the expected value of the determinant of the Fisher information matrix:

$$\hat{\boldsymbol{k}}_m, \hat{\boldsymbol{d}}_m = \arg\min_{\boldsymbol{k}_m, \boldsymbol{d}_m} \mathbb{E}\left[-\log\left(\det\left\{\left(\mathcal{I}(a, d_h; \mathbf{k}_m, \mathbf{d}_m)\right)\right\}\right)\right] \qquad (6)$$

where the hat symbol denotes the set of parameters that produce an optimal cost function.

An illustrative depiction of the optimization process of the cost function $Q(a, d_h; \mathbf{k}_m, \mathbf{d}_m)$ is shown next. The optimized surface exhibits a well-defined minimum which facilitates fast and accurate convergence.

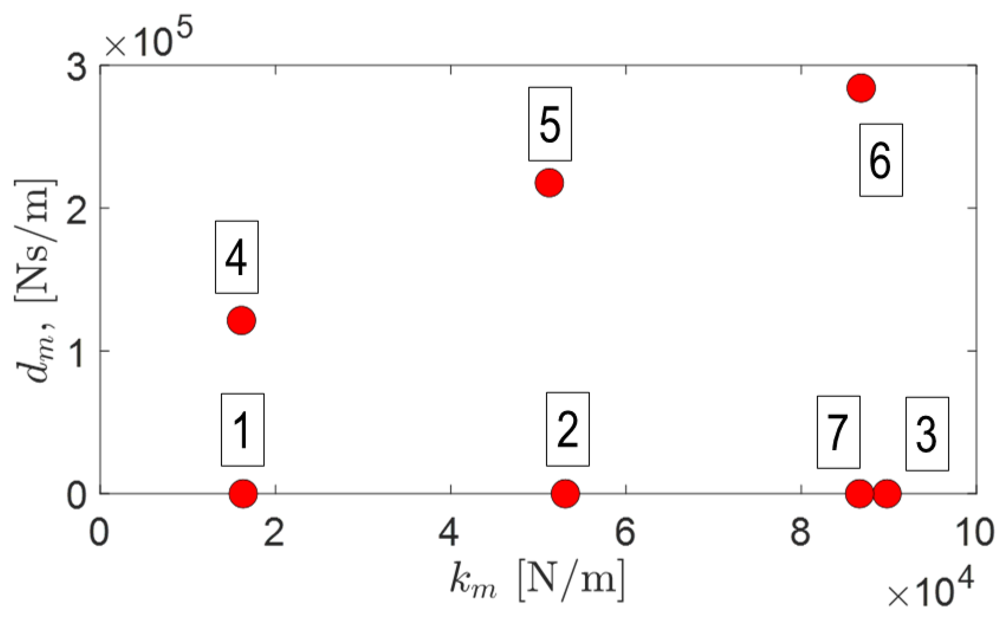


The experimental design procedure was used for determining an optimal experimental setup in the CYBERLAB campaign. The procedure indicated that the following mooring parameters are optimal and would lead to most informative data:

$$\hat{\boldsymbol{k}}_m = [16, 90]\ \mathrm{kN/m}$$
$$\hat{\boldsymbol{d}}_m = [0, 0]\ \mathrm{kNs/m}$$

Experimental Results

The proposed procedure was validated with experimental data from 7 experiments leading to 21 possible design pairs. The possible pairs are shown in the table on the right with [1, 3] being the optimal design. The parameter space of the experiments is shown below,

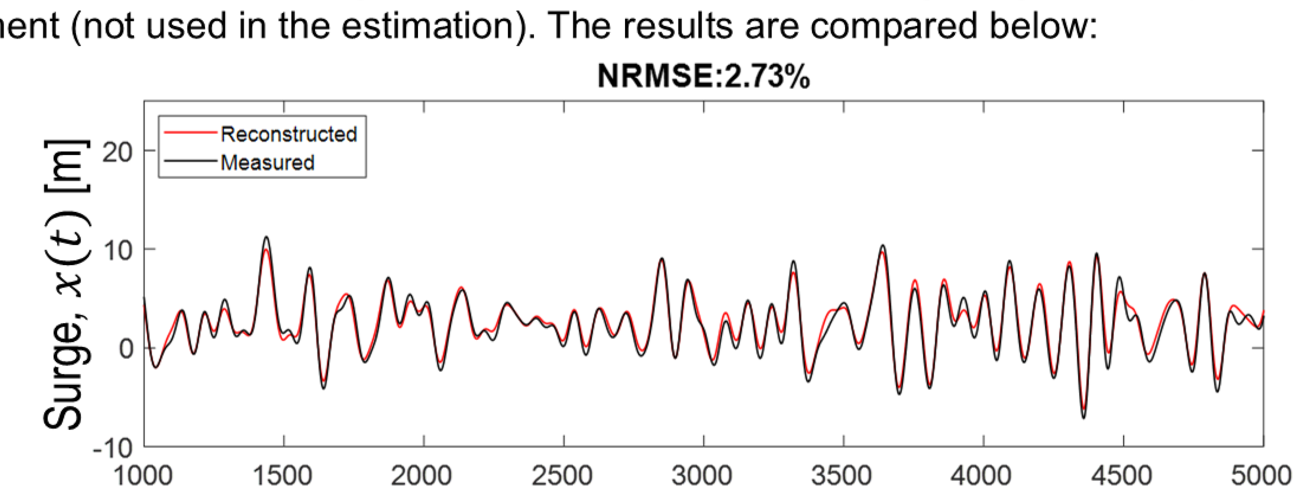


First, all 7 experiments and Eq. (4) were used to establish a ground truth for the LF added mass and damping of the:

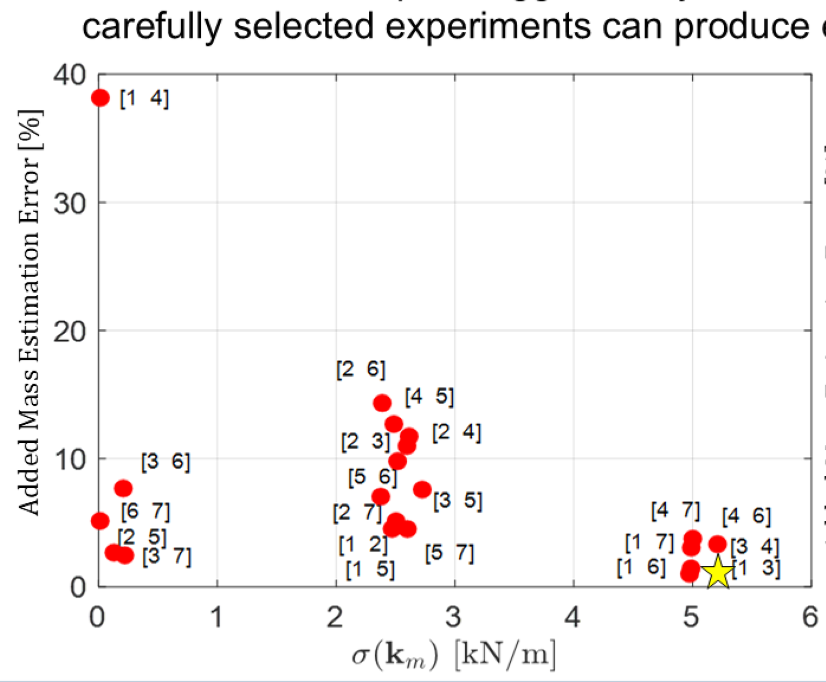
Assumed Ground Truth:
$\hat{a} = 8\,751\,000$ kg
$\hat{d}_h = 645$ kNs/m

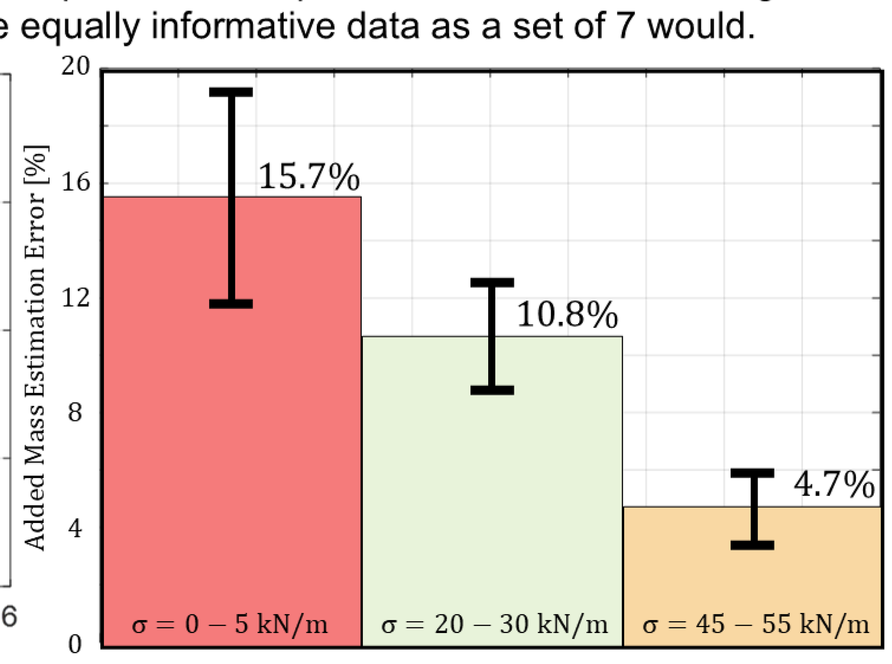
With the estimated added mass and damping the LF force $f_\omega(t)$ is easily computed from (1).

The reconstructed force signal is then used for time-integrating the response of a new experiment (not used in the estimation). The results are compared below:



The estimation based on the design pair is compared to the ground truth. The performance is shown below. The pair suggested by the OED procedure outperformed the rest indicating that 2 carefully selected experiments can produce equally informative data as a set of 7 would.





Experimental Pairs		$\sigma\{k_m^{(1)}, \ldots, k_m^{(N)}\}$ (kN/m)
Exp (1)	Exp (2)	
3	4	52.1
☆ 1	3	52.0
4	6	50.9
1	6	50.7
4	7	48.0
1	7	47.9
3	5	27.1
2	4	26.1
1	2	26.0
2	3	26.0
5	6	26.0
4	5	24.9
1	5	24.8
2	6	24.7
5	7	23.1
2	7	21.9
3	7	4.1
6	7	2.9
2	5	1.2
3	6	1.2
1	4	0.1

AARHUS UNIVERSITET · CYBERLAB KPN · EERA DeepWind CONFERENCE 2025

This research has been funded by the Research Council of Norway through project 326654 CYBERLAB KPN, a collaboration between SINTEF Ocean, NTNU, Aarhus University, Equinor, Mainstream Renewable Power, APL Norway, Sevan Deepwater Technology and Delmar Systems.