# Prediction of vertical profiles of offshore wind considering the effects of atmospheric stratification

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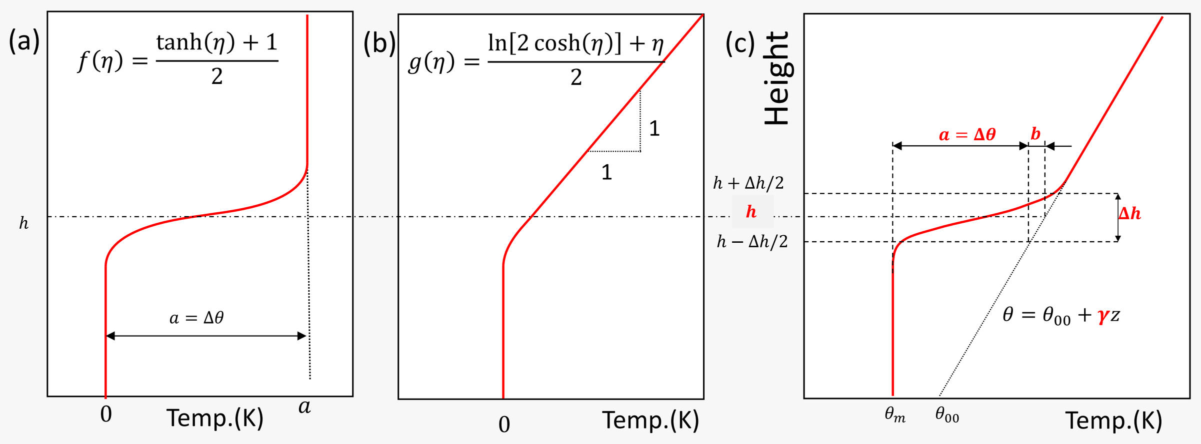
## Introduction

The prediction of turbine wake behavior in offshore wind farms requires precise evaluation of vertical inflow profiles, especially as larger turbines operate at heights where wind profiles are highly sensitive to atmospheric stratifications. The IEC 61400-1[1] standard recommends a wind shear exponent $\alpha = 0.14$, while observations reveal significant variability due to upper and lower atmospheric stratification, which affect turbulent mixing and wind speed distributions as shown in study by Kikuchi et al. (2022)[2]. Monin-Obukhov similarity theory (MOST) widely used for wind profile prediction performs well in the surface layer under 100m but overestimates wind speeds above 100m under stable stratified conditions.

In this study, a three-layer potential temperature model is used to identify the key parameters in Choshi , Japan, a combined wind speed profile model is proposed by considering upper and lower atmospheric stratification and validated using LiDAR measurement at Choshi.

## Evaluating upper-atmospheric stratification parameters

### ❑ Potential temperature model




$$\theta(z) = \theta_m + af(\eta) + bg(\eta) \quad (1)$$

$$f(\eta) = \frac{tanh(\eta) + 1}{2}$$

$$g(\eta) = \frac{ln[2\,cosh(\eta)] + \eta}{2}$$

$$\eta = 3(z - h)/(\Delta h)$$

$$\gamma = 2b/\Delta h$$

*Figure 1. three-layer potential temperature model*

**Table 1. List of functions and parameters**

| | |
|---|---|
| $f(\eta)$ | Function of capping-inversion layer (Figure 1a) |
| $g(\eta)$ | Function of mixed layer (Figure 1b) |
| $h$ | Capping-inversion height (m) |
| $\Delta h$ | Capping-inversion layer thickness (m) |
| $\theta_m$ | Potential temperature of the mixed layer (K) |
| $\Delta\theta$ | Capping-inversion strength (K) |
| $\gamma$ | Lapse rate (K/km) |

The three-layer (constant potential temperature mixed layer, a strongly stratified entrainment layer and a constant lapse rate free atmosphere) potential temperature model proposed by Rampanelli et al. (2004)[3] is used to identify the vertical potential temperature distribution and its parameters. Excluding the surface layer, the three-layer structure is expressed as the vertical distribution of potential temperature $\theta(z)$ (equation (1)).

### ❑ Parameters identification

Based on the study by Rampanelli et al. (2004)[3] which applied parameter identification to observation data. However, the rules for identification were not thoroughly discussed in previous study. To identify given profile $\theta_i(z)$ with range $z \in (z_{min}, z_{max})$, following three rules are proposed for robust identification:

***1. Extrapolation of $\theta$ at $z = 0$ :*** Since 3 layers model excludes the surface layer, ground temperature cannot be directly included in the objective function. Instead, the error of mixed-layer temperature $\theta_m$ included into objective function $RMSE = \sqrt{\left[\sum_{z=z_{min}}^{z=z_{max}}\left(\theta(z) - \theta_i(z)\right)^2 + k\left(\theta_m - \theta_i(z_{min})\right)^2\right]/(n+k)}$. where $n$ is point numbers of data. $k$ is number of extrapolation points. With the extrapolation *of* $\theta$ *at* $z = 0$, error of $\theta_m$ is reduced as shown in Figure 2a.

***2. Constraints for h and Δh:*** For a given range $z \in (z_{\min}, z_{max})$, $h$ and $\Delta h$ must be smaller than $z_{max}$. As shown in Figure 1c, to capture the third layer above $h + \Delta h/2$, $h$ *and* $\Delta h$ should satisfy $h < \alpha z_{max}$ and $\Delta h/2 < \alpha z_{max}$. When $\alpha = 1$ the upper layer cannot be captured thus $\gamma = 0$ occurs. After adopting $\alpha = 2/3$, case of $\gamma = 0$ is vanished as shown in Figure 2b.

***3. Estimation of h and Δh using iterative method:*** Compare with the study at North Sea, Europe by Lanzilao et al, (2024)[4]. Japan's atmosphere is more unstable, with higher capping inversion layers. The 3000m range used for the North Sea fails to capture the actual $h$ *and* $\Delta h$ in Japan. In this study, the $z_{max}$ start from 3000m and increases when $h$ and $\Delta h$ hit upper limit until $z_{max} = 6500m$. After adopting proposed iterative method, the influence due to height limit is vanished, $\Delta h$ successfully identified as shown in Figure 2c.

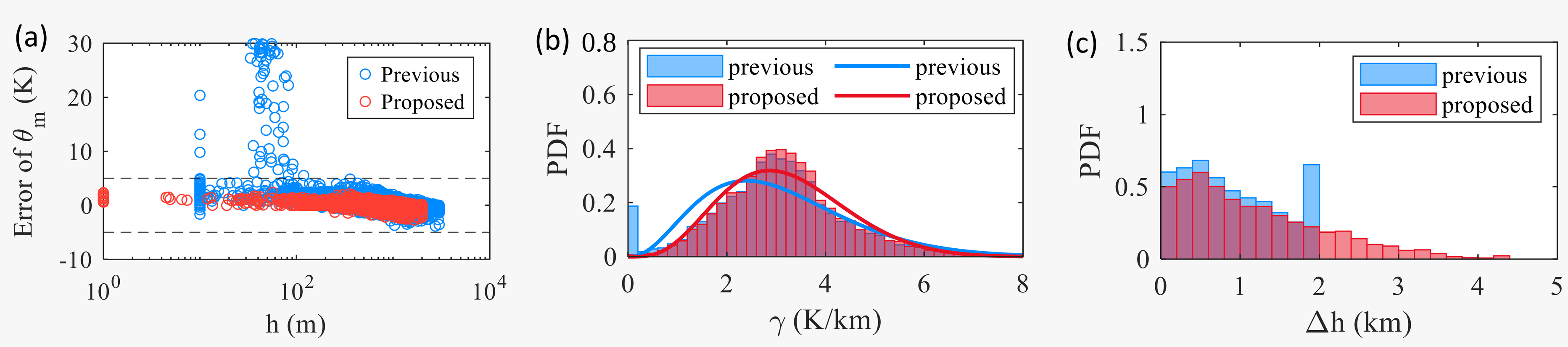



*Figure 2. (a) relationship between identified $h$ and error of $\theta_m$, (b) distribution of identified $\gamma$, (c) distribution of identified $\Delta h$.*

### ❑ Comparison of identified parameters with north sea

Parameters is identified based on WRF Simulation at Choshi offshore test facility(Feb 2013–Jan 2014)[2]

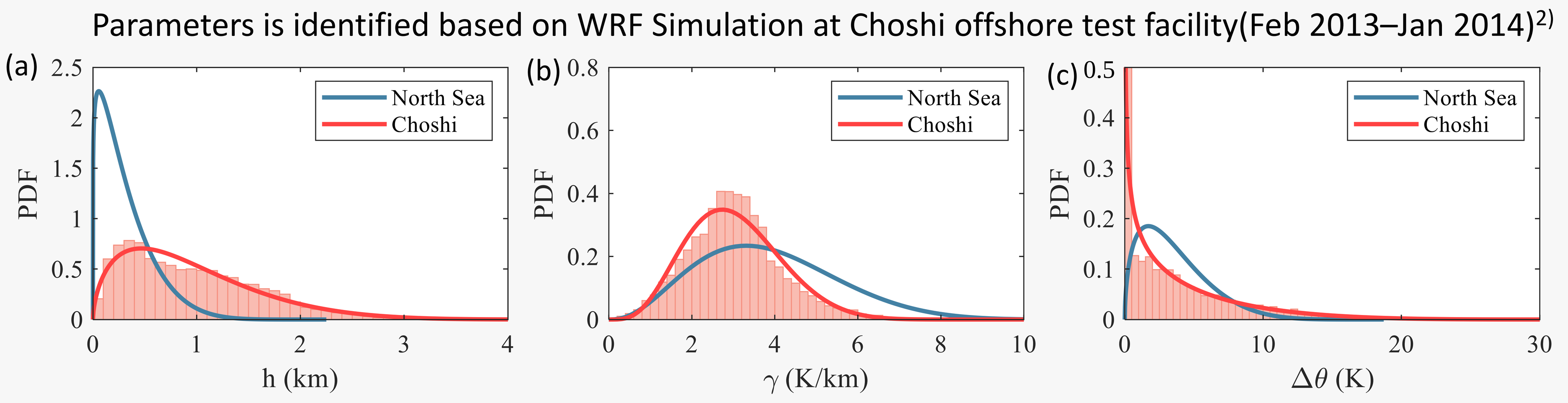



*Figure 3. Distribution of identified parameters,(a) Capping-inversion height $h$,(b) Lapse rate $\gamma$,(c) Capping-inversion strength $\Delta\theta$.*

From a comparative analysis on the identified key parameters between Choshi, Japan and North Sea, Europe[4], it is observed that the capping-inversion height $h$ is distributed over a wider range, mean lapse rate $\gamma$ is lower and the capping-inversion strength $\Delta\theta$ is weaker in Japan as shown in Figure 3.

## Conclusions

- The parameters in potential temperature model are identified using the proposed method and show reasonable distribution. The results in Choshi, Japan indicate a lower mean lapse rate $\gamma$ and higher occurrence of the mixed layer than those in North Sea, Europe.
- A combined wind speed profile model considering upper and lower atmospheric stratification is proposed and shows good agreement with LiDAR measurement across unstable, neutral, and stable conditions.

## Modeling vertical profile of the offshore wind

### ❑ Influence of the upper-atmospheric stratification on the wind profile

To evaluate the Influence of the upper-atmospheric stratification on the wind profile, analytical model proposed for Conventional Neutral Boundary Layer (CNBL) by Liu et al. (2022)[5] is used in this study as:

$$u_{CNBL}(z) = \frac{u_*}{\kappa}\left[ln\xi + a(Zi)(1-\xi) + a_\psi(Zi)\psi + lnb(Zi) + ln(h'/z_0) - A(Zi)\right] + u_g \quad (2)$$

$$v_{CNBL}(z) = \frac{u_*}{\kappa}\left[-\xi B(Zi) + b_\psi(Zi)\psi\right] + v_g \quad (3)$$

where $u^*$ is friction velocity, $\kappa = 0.4$ is von Kármán constant, $\xi = z/h$ is a dimensionless parameter, $h' = h/\left(1 - 0.05^{2/3}\right)$ is boundary layer depth. $Zi = \sqrt{\gamma g/\theta_0}/f_c$ is Zilitinkevich number, $\gamma$ is lapse rate,$g = 9.85\ m/s^2$ is gravitational acceleration, $\theta_0 = 300K$ is reference potential temperature, $f_c$ is Coriolis's coefficient. $u_g$ and $v_g$ are u and v components of geostrophic wind speed $G$. $a$, $a_\psi$, $b$, $b_\psi$, $A$ and $B$ can be determined as:

$$a(Zi) = A(Zi) - lnb(Zi),\ b(Zi) = \left(1 - 0.05^{2/3}\right)\beta(Zi),\ \beta(Zi) = \left(C_R^{-2} + C_N^{-2}Zi\right)^{1/2}$$

$$a_\psi(Zi) = \frac{2 - a(Zi)}{1 - 2\epsilon},\ b_\psi(Zi) = \frac{2\kappa b(Zi) - B(Zi)}{1 - 2\epsilon},\ \epsilon = \frac{h' - z_i}{2h'},\ \psi(\xi) = \xi - \frac{e^{\xi/\epsilon} - 1}{e^{1/\epsilon} - 1}$$

$$A(Zi) = -A_1 m(Zi) + ln\left(A_0 + m(Zi)\right) + ln\beta(Zi),\ B(Zi) = \left(B_0 + B_1 m(Zi)^2\right)\beta(Zi)^{-1}$$

$$m(Zi) = \left(1 + C_m^2 Zi^2\right)^{1/2}\beta(Zi)^{-1}$$

where $A_1 = 0.65, A_0 = 1.3, B_1 = 7, B_0 = 8, C_m = 0.1, C_R = 0.5, C_N = 1.6, \epsilon = 0.12$, and $z_i = 0.76h'$.

Four parameters are used to predict vertical profile with upper-atmospheric stratification: $f_c$, $z_0$, $\gamma$ and $G$. Iterative calculations used to determine $u^*$ and $h$.

### ❑ Influence of lower-atmospheric stability on the wind profile

At wind turbine hub heights, the influence of lower-atmosphere stability must be considered. Therefore, stability corrections is applied based on the Monin-Obukhov Similarity Theory (MOST) proposed by Yamaguchi et al. (2024)[6] is applied following method:

$$U_{target}^{pred}(z) = U_{ref}^{obs} \times C_U^{terrain}(z) \times C_U^{Stability}(z) \times C_U^{CNBL}(z) \quad (4)$$

$$C_U^{stability}(z) = \frac{\left[ln\left(\frac{z}{z_0}\right) - \psi\left(\frac{z}{L(x,y,t)}\right)\right] / \left[ln\left(\frac{z_{ref}}{z_0}\right) - \psi\left(\frac{z_{ref}}{L(x,y,t)}\right)\right]}{\left[ln\left(\frac{z}{z_0}\right)\right] / \left[ln\left(\frac{z_{ref}}{z_0}\right)\right]}$$

$$C_U^{CNBL}(z) = \frac{U_{CNBL}(z)}{U_{CNBL}(z_{ref})} = \frac{\sqrt{u_{CNBL}^2(z) + v_{CNBL}^2(z)}}{\sqrt{u_{CNBL}^2(z_{ref}) + v_{CNBL}^2(z_{ref})}}$$

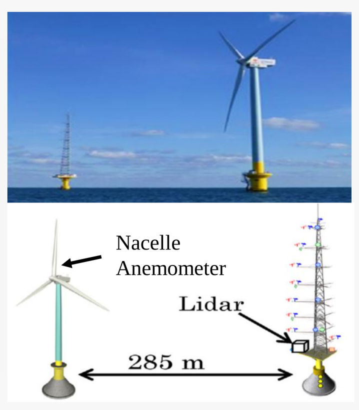



*Figure 4. Choshi offshore wind energy test facility.*

where, $U_{ref}^{obs}$ is observed wind speed at nacelle anemometer as shown in Figure 4, $C_U^{terrain}(z)$ accounts for terrain effects and $C_U^{stability}(z)$ for stability. Since this study focuses on offshore condition, terrain effect correction factor $C_U^{terrain}(z)$ is set to 1. $C_U^{CNBL}(z)$ is the correction of upper-atmospheric stratification calculated based on equation (2) and (3).

In this study, the range of stability correction extend up to $3z_{ref}$ under unstable and neutral conditions. The height of the surface boundary layer is lower under stable conditions. Thus, stability corrections are applied only up to $z_{ref}$ as shown in Figure 5.

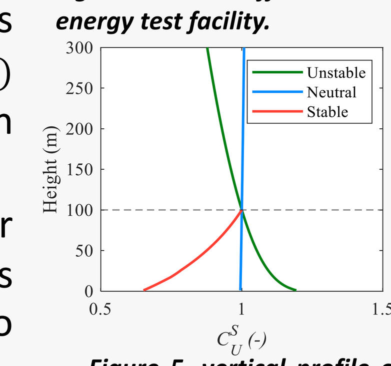



*Figure 5. vertical profile of stability correction factor*

### ❑ Validation of predicted wind profile using LiDAR measurements at Choshi

This study uses $L$ from WRF Simulation since $L$ from WRF Simulation agrees well with observation.[2]

For Choshi offshore condition, $f_c = 8.45 \times 10^{-5}$, $z_0 = 0.0002m$ are used.

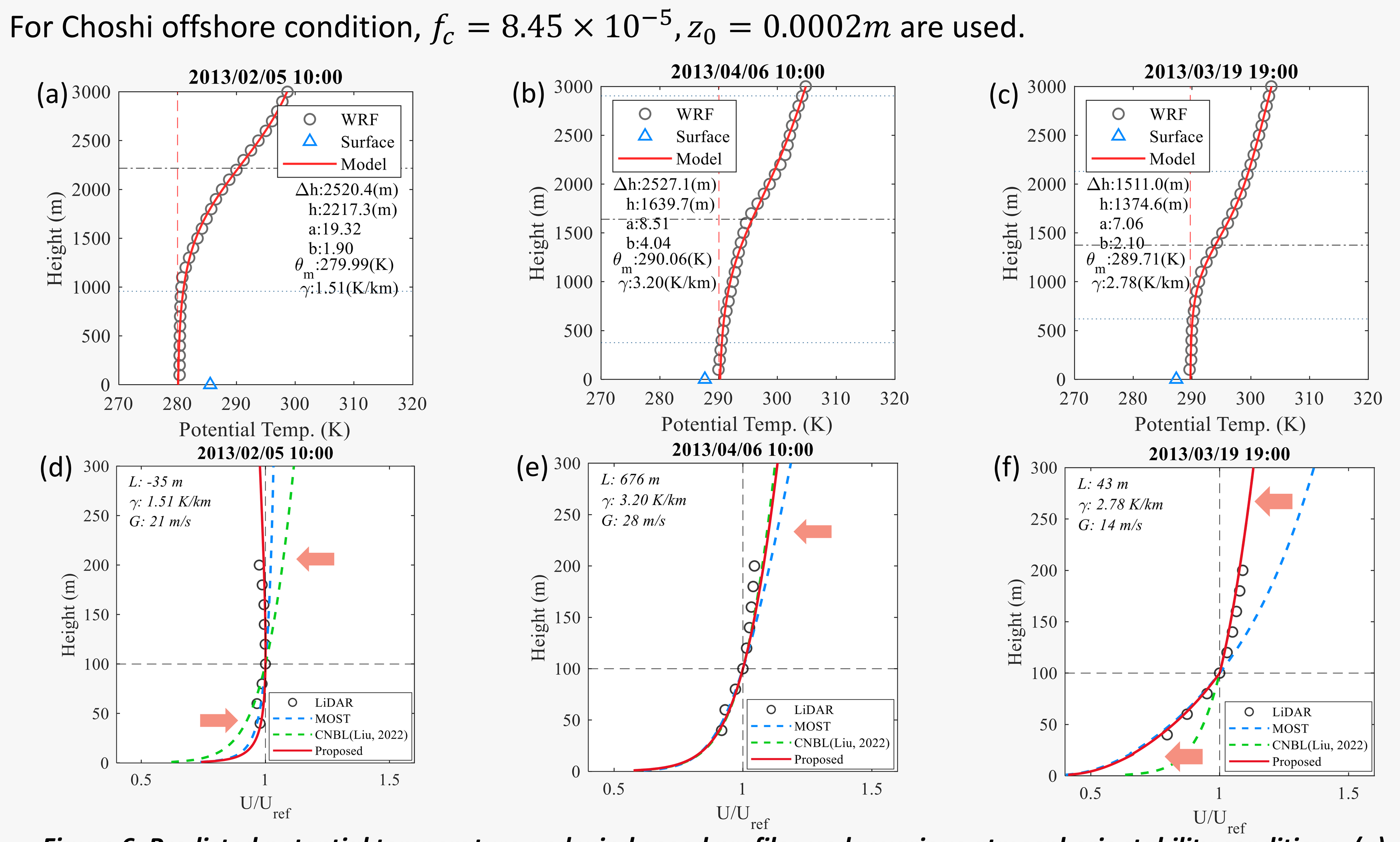



*Figure 6. Predicted potential temperature and wind speed profiles under various atmospheric stability conditions: (a) and (d) represent unstable conditions, (b) and (e) represent neutral conditions, (c) and (f) represent stable conditions.*

In unstable conditions, the proposed model improves prediction accuracy due to lower-atmospheric stability. In neutral conditions, it also shows better accuracy due to upper-atmospheric stratification. In stable conditions, the overestimation of CNBL model under 100m is improved by considering to lower-atmospheric stability, overestimation of MOST model above 100m is improved by considering upper-atmospheric stratification as shown in Figure 6.

## Acknowledgement

This research is carried out as a part of research and development on observation and evaluation of wind turbine wakes found by the New Energy and Industrial Technology Development Organization (NEDO). The authors wish to express their deepest gratitude to the concerned parties for their assistances.

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