A Process Oriented Modeling Concept for Rich Vehicle Routing Problems

Andreas Reinholz

VIP'08
13.06.2008

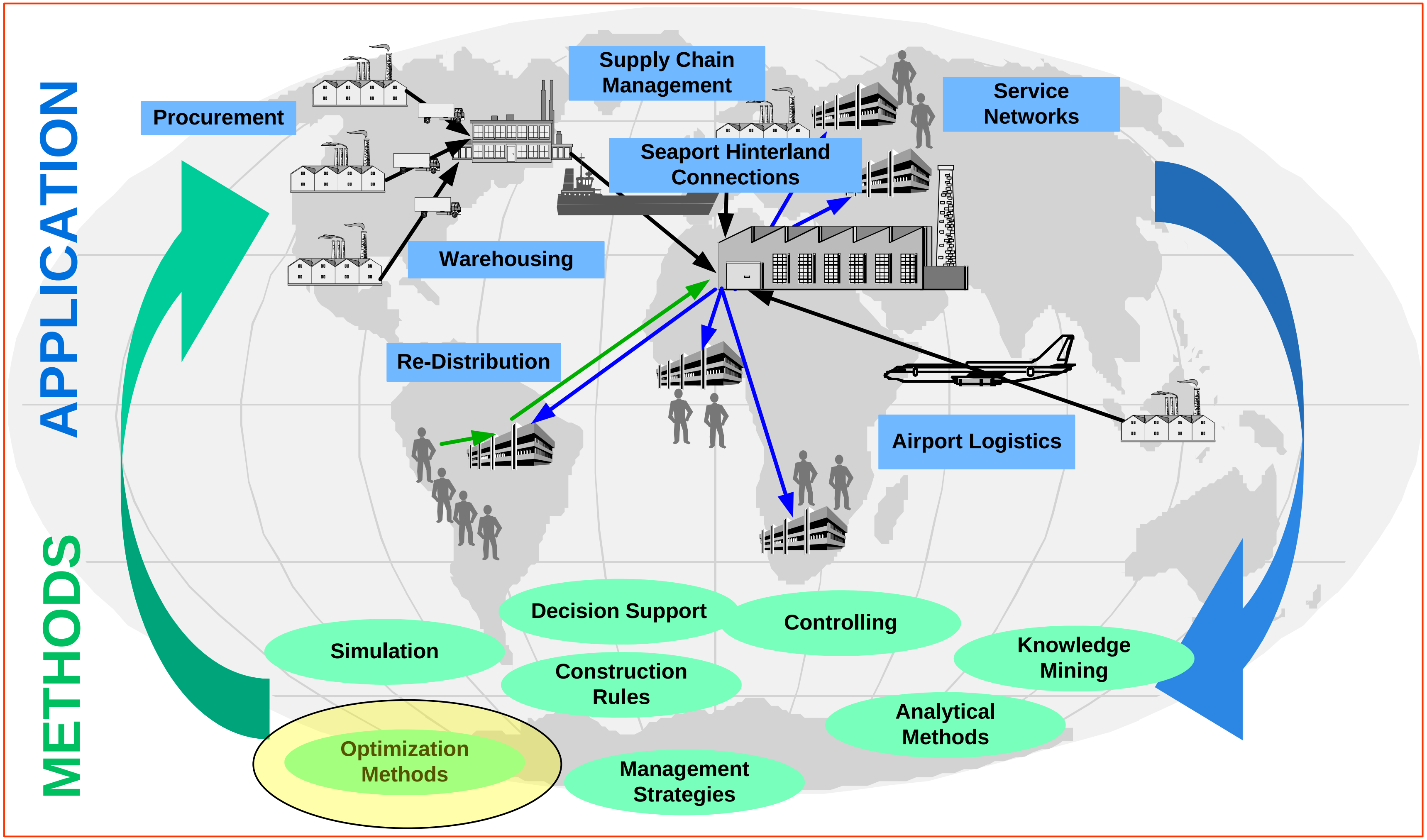
Algorithm Engineering
Technical University of Dortmund

andreas.reinholz@gmx.de

Structure

- Motivation and context
- Metaheuristics as Iterative Variation Selection Procedures
- Elementary and composed Neighborhood Generating Operators
- Informal problem description of Vehicle Routing Problems (VRP)
- Modeling concepts and constraint handling
- Neighborhood Generating Operators for Vehicle Routing Problems
- Acceleration techniques and efficient data structures
- Decomposition methods
- Closer to the real world: Modeling uncertainty, flexibility and risk
- Conclusions and outlook

Projects of the SFB559



Features and Challenges of Logistic Optimization Tasks

Mixed – Integer Optimization Problems with

- Various **constraints**
- **Multiple objectives**
- Range from **Strategic Planning** to **Online-Optimization**
- **Open** or **Disturbed Systems**, imprecise or incomplete data, noise
- **Dynamic Optimization** tasks with moving optima
- **Hierarchies** of complex optimization problems
- Integration in “**Interactive Decision Support Systems**”
- Evaluation model could be a **Simulation Model** or a “**Black Box**”

Metaheuristics

- Neighborhood Search (NS)
- Variable Neighborhood Search (VNS)
- Iterative Local Search (ILS)
- (Recursive) Iterative Local Search (R-ILS)
- Tabu Search (TS)
- Greedy Randomized Adaptive Search Procedure (GRASP)
- Evolutionary Algorithms (EA)
- Ant-Systems, Particle Swarm, …
- Scatter Search
- Adaptive Memory Programming
- Estimation of Distribution Algorithms (EDA)
- Multiple Agent Systems
- Stochastic Local Search (SLS)
- …

The 10 commandments for powerful Hybrid Metaheuristics

1. I'm the concept of Hybrid Metaheuristics your preferred optimization method, who brought you out of the land of exact methods, out of the house of slavery.
2. …
3. …
4. …
5. …
6. …
7. …
8. …
9. …
10. You shall covet your best competitors procedures, methods and strategies, break them into parts and use them as Local Search.

Scheme of an Iterative Variation Selection Procedure (IVS)

Initialization
…
REPEAT
    Select Candidate Solutions for Modification
    …
    Modify Candidate Solutions
    …
    Select Candidate Solutions for further Iterations
    …
UNTIL Stopping Criteria(GNr, LastImprovingGNr, Threshold, …);

IVS: Horizontal and Recursive Composition

```
IVS( RecLevel, NS_Set, IVS_ParaSet )
Initialization
…
REPEAT
        FOR (HLevel = 0) TO GetMaxHLevel(…) DO
                Select Candidates for Modification
                Modify Candidates
                IVS( RecLevel-1, NS_Set, IVS_ParaSet )
                …
                Select Candidates for further Iterations
UNTIL Stopping Criteria( GNr, LastImprovingGNr, Threshold, Level… );
```

Design of Modeling and Optimization Components

Tasks for the designer of the optimization problem

- Develop an **Evaluation Model**
  - Mathematical or algorithmic **description of the search space** (e.g. decision variables)
  - Definition of meaningful **quality criteria** and **objective functions**
  - Description of the **constraints**
  - Definition of **penalty functions**
- Provide consistent **input and test data** for modeling and optimization

Tasks for the designer of the optimization procedures

Tasks for the designer of the optimization procedures

- Develop a **coding** of the search space
- Develop **variation operators**, that generate candidate solutions from already available solutions and integrate them into **Metaheuristics**
- Define **fitness functions** out of
  - **quality criteria** and **objective functions**
  - **penalty terms**
  - and additional **search control terms**
- Determine suitable **parameters** for the designed Metaheuristics

Variation Operators

Systematic modification of decision variables

- **Deterministic** principals
- **Stochastic** principals
- **Local** view (i.e. modify only few variables at each step)
- **Global** view (i.e. Tree Search)
- **Construction**, **destruction** or **modification** schemes
- **Decomposition** strategies (hierarchical, geographical, functional)
- **Combined** or **composed** variation operators (i.e. VNS, Mutation)

Neighborhood Generating Operators

- Elementary Neighborhood Generating Operator =
  Systematic parameterized modification of decision variables
  - One Step Neighborhood
  - Neighborhood Transition Graph (NH-Transition Graph)
  - (Asymmetric) Distance measure, metric
- Neighborhood Search templates
  - Steepest ascent
  - Next ascent
- K-Step Neighborhood
  - Local optima of quality K (iterative or recursive scheme)
  - Discrepancy Search, Local Branching
  - Rapid-Tree Search, Rapid-B&B
  - Probabilistic K-Step Neighborhood (i.e. Mutation-Operator)

Multiple Solution Variation Operators (Recombination)

- Recombination Operator =
  Parents define a subspace or a subset of the search space
  - Standard Crossover = **Randomly selected point** in this subset
  - **Series of points** in this subset using a NH-Transition Graph
    - **Deterministic** principles
      - **Connecting path** between parents (with discrepancies)
      - **Enumerate** the complete subset
      - **Deterministic Sub-Problem Solver**
    - **Probabilistic** principles
      - **Re-Sampling** or **Random Walk**
      - **Connecting random path** (with discrepancies)
      - **Probabilistic Sub-Problem Solver**

Combining Neighborhoods

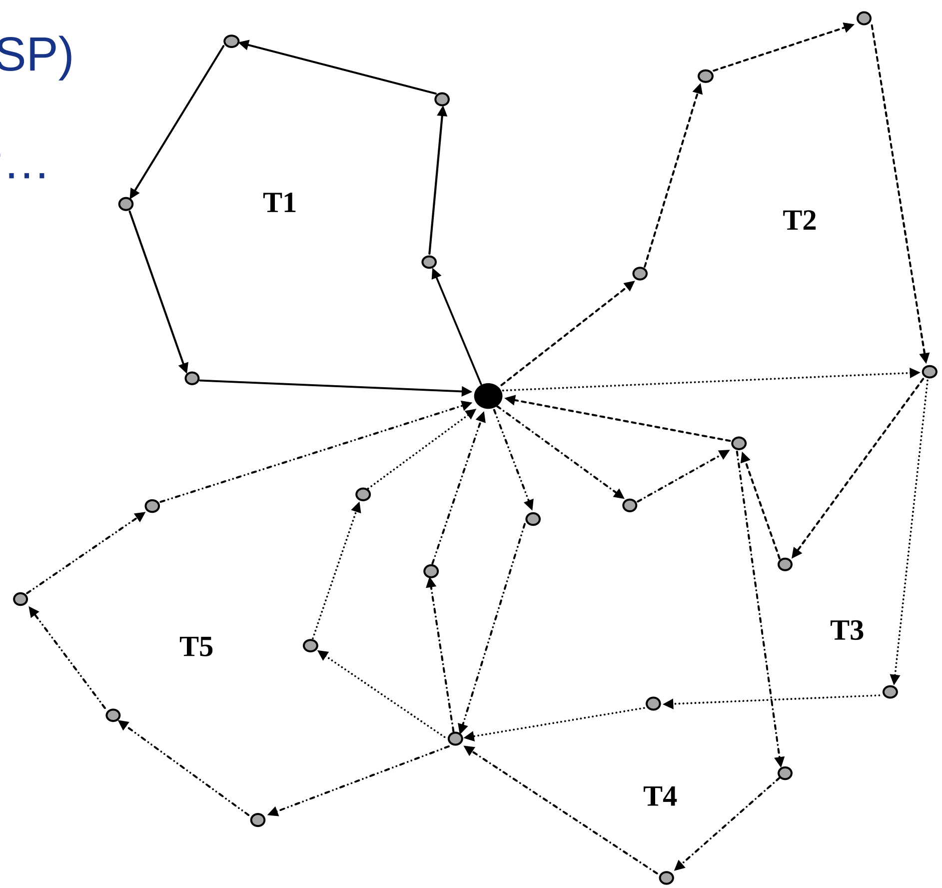
- Variable Neighborhood Search
  - Fixed sequence
  - Probabilistic sequence
  - Adaptive or self-adaptive
- Evolutionary Algorithms
  - Mutation Operator (Probabilistic K-Step Neighborhood)
  - Crossover Operator (Dynamic Sub Problem Search)
- Hybrid Evolutionary Algorithms
  - i.e. Hybrid (1+1) EA = Iterative Local Search
- Multi - Start Metaheuristics
  - Number of runs vs. number of iterations (Multi Start Factor)

Aspects of Iterative Variation Selection Procedures

- Problem specific representation
- Problem specific variation operators
- (Variable) Neighborhood Search techniques
- Accelerated Delta Evaluation of the objective function
- Efficient data structures
- Dynamic Adaptive Decomposition strategies (DADs)
- Biased disruption strategies
- Adaptive or self-adaptive search control
- Population Management

Vehicle Routing Problems (VRP)

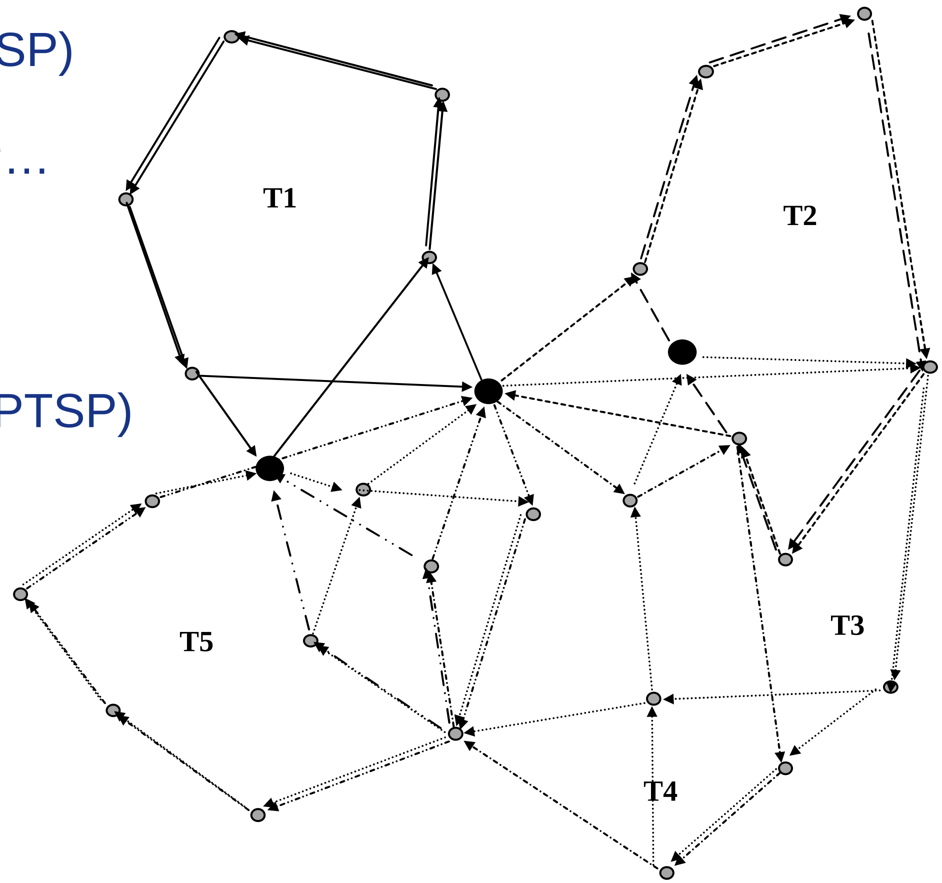
- "Traveling Salesman Problem" (TSP)
- CVRP, VRPTW, VRPBH, PDVRP…
- "Open VRP" (OVRP)



Vehicle Routing Problems (VRP)

- "Traveling Salesman Problem" (TSP)
- CVRP, VRPTW, VRPBH, PDVRP…
- "Open VRP" (OVRP)

- "Periodic TSP and VRP" (PVRP, PTSP)
- "Multiple Depot VRP" (MDVRP)
- "Periodic MDVRP" (PMDVRP)

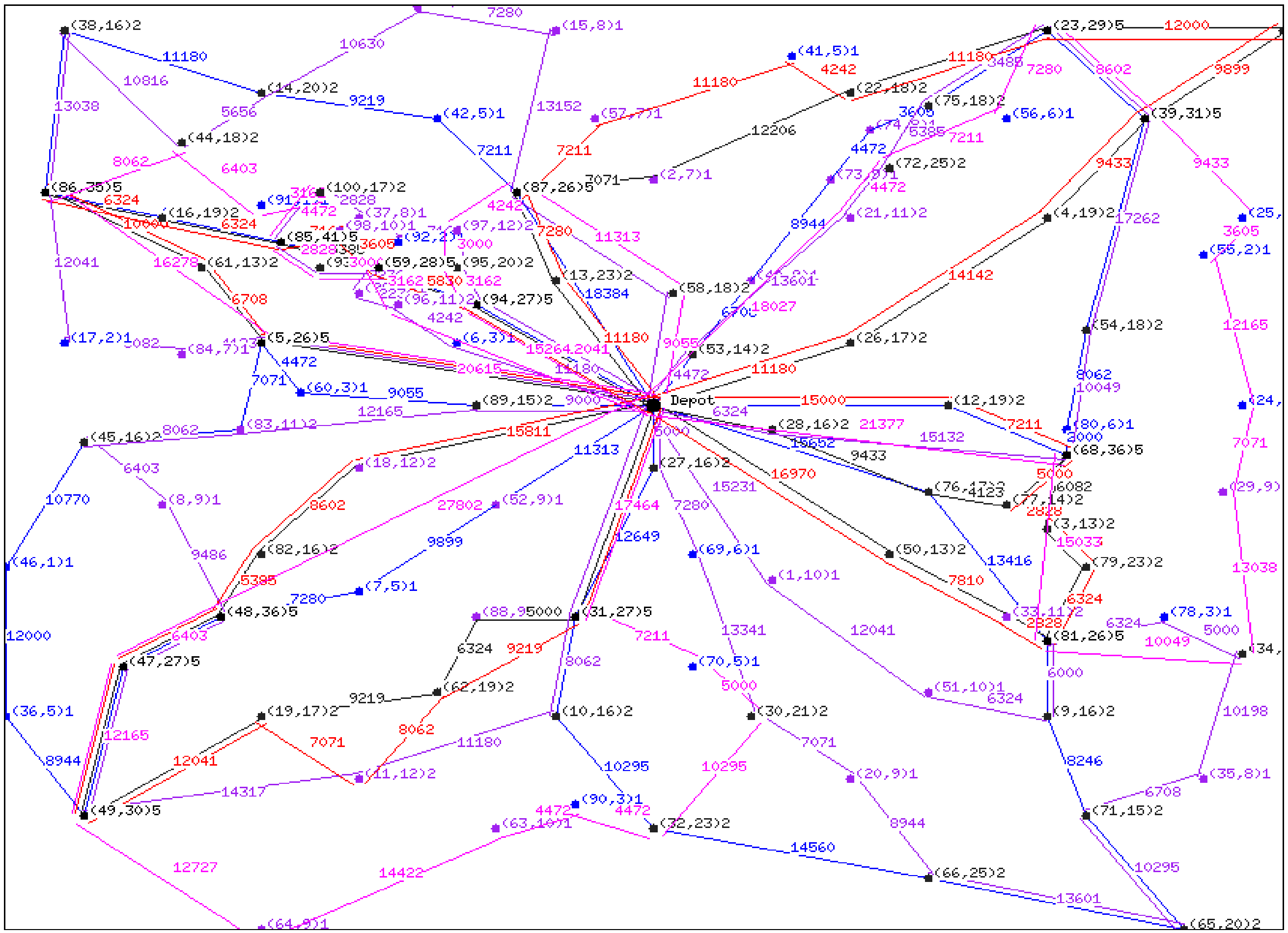


Nr	Total	0	1	2	3	4	
TourL	2031736	423970	345711	522865	353335	385855	
TourLC	2031736	423970	345711	522865	353335	385855	
Costs	2031736	423970	345711	522865	353335	385855	
TourLEu	2031.82	423.98	345.73	522.89	353.35	385.87	
TourLEuF	2031.82	423.98	345.73	522.89	353.35	385.87	
TourLEUI	1957	408	332	504	339	374	
TourLEUIT	1787	376	304	455	312	340	
VU	3938	776	742	979	750	691	
TourCount	21	4(-1)	4(-1)	5	4(-1)	4(-1)	
D2Best	2034.15	2031.82	0.11%	2.33	VU ok	TL ok	TNr ok : 0

TNr ok : 0

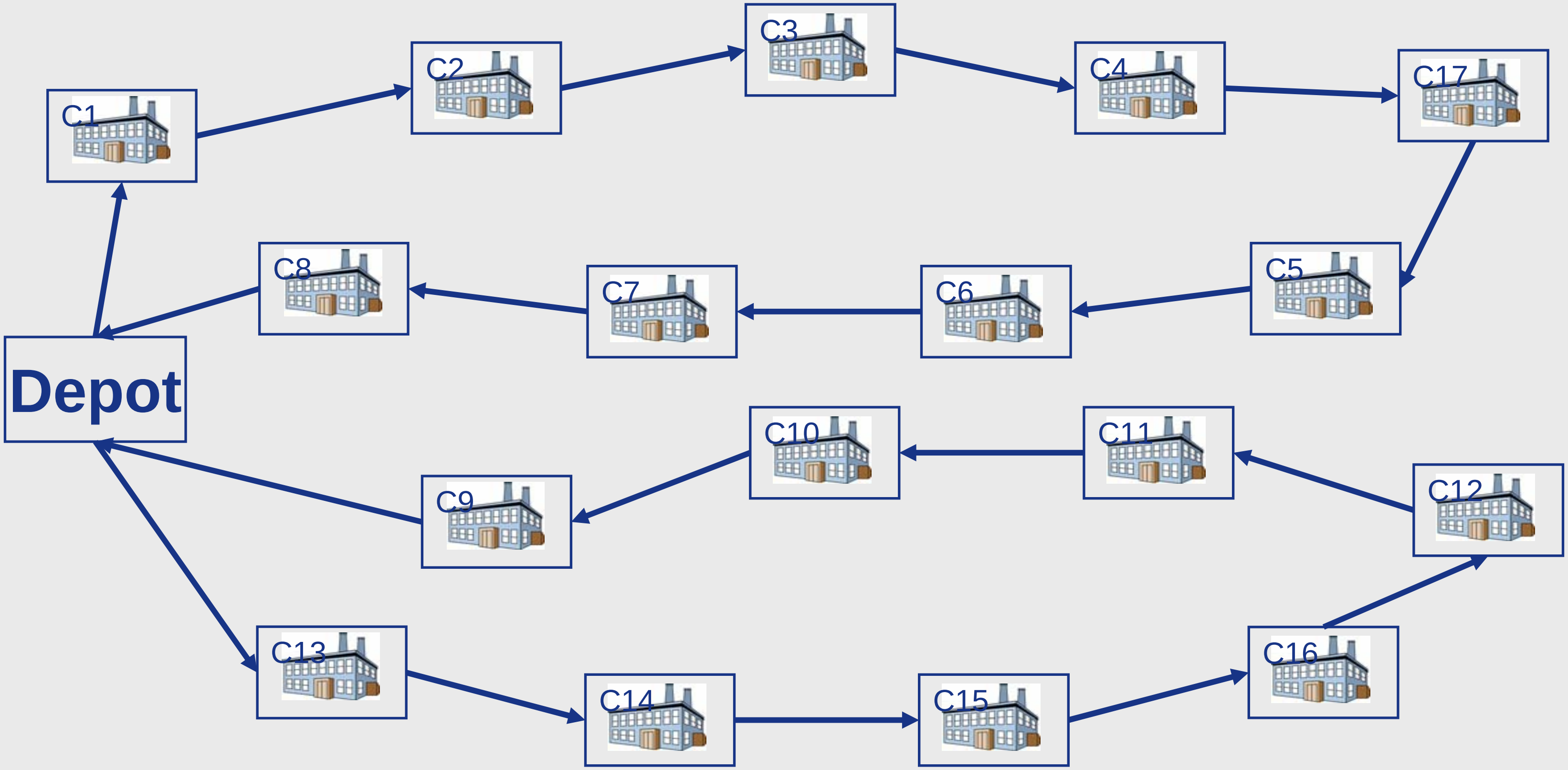
Operations : 461693

File | Parameter | Read | Mouse | Display | SAVE | RESTORE | Mo | Tu | We | Th | Fr | Sa | Su | D7 | D8 | D9 | SortDist | SortDist1



Modeling Concepts

- Resource Consumption Concept, Set of Resources
- Finite State Machines



Examples for Constraints and Modeling Aspects

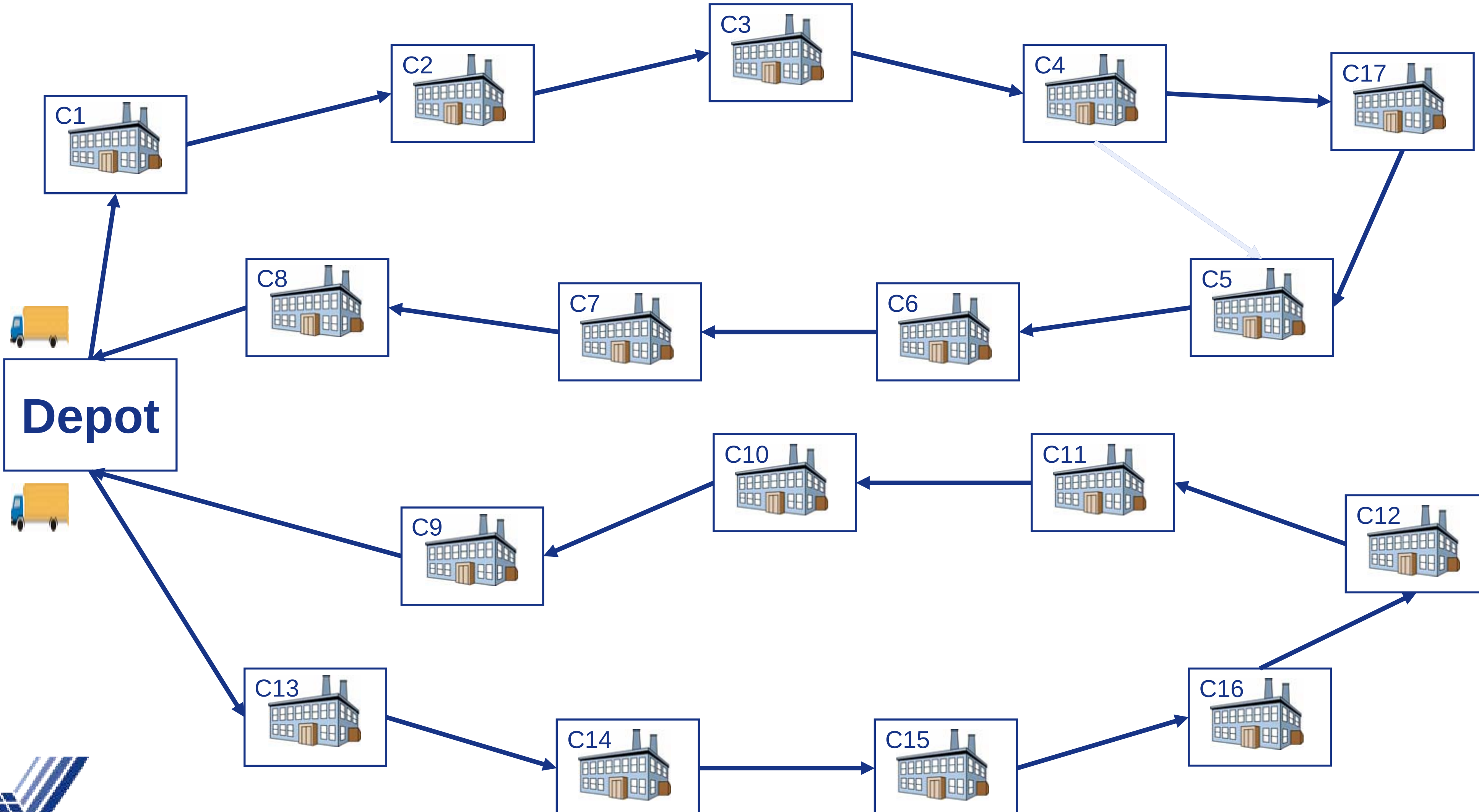
- Capacity limit
- Tour length limit
- Time Windows
- Split Demand, Single Unit VRP
- Pickup and Delivery
- Backhauls
- Heterogeneous fleet
- Multiple compartments, dynamic compartment sizes, load restrictions
- Fixed costs
- Customer dependent costs
- Asymmetric distance and driving costs
- Customer specific service times, back on route times
- Traffic flow factor
- Flexible starting times

Operators / Neighborhoods / Neighborhood Size

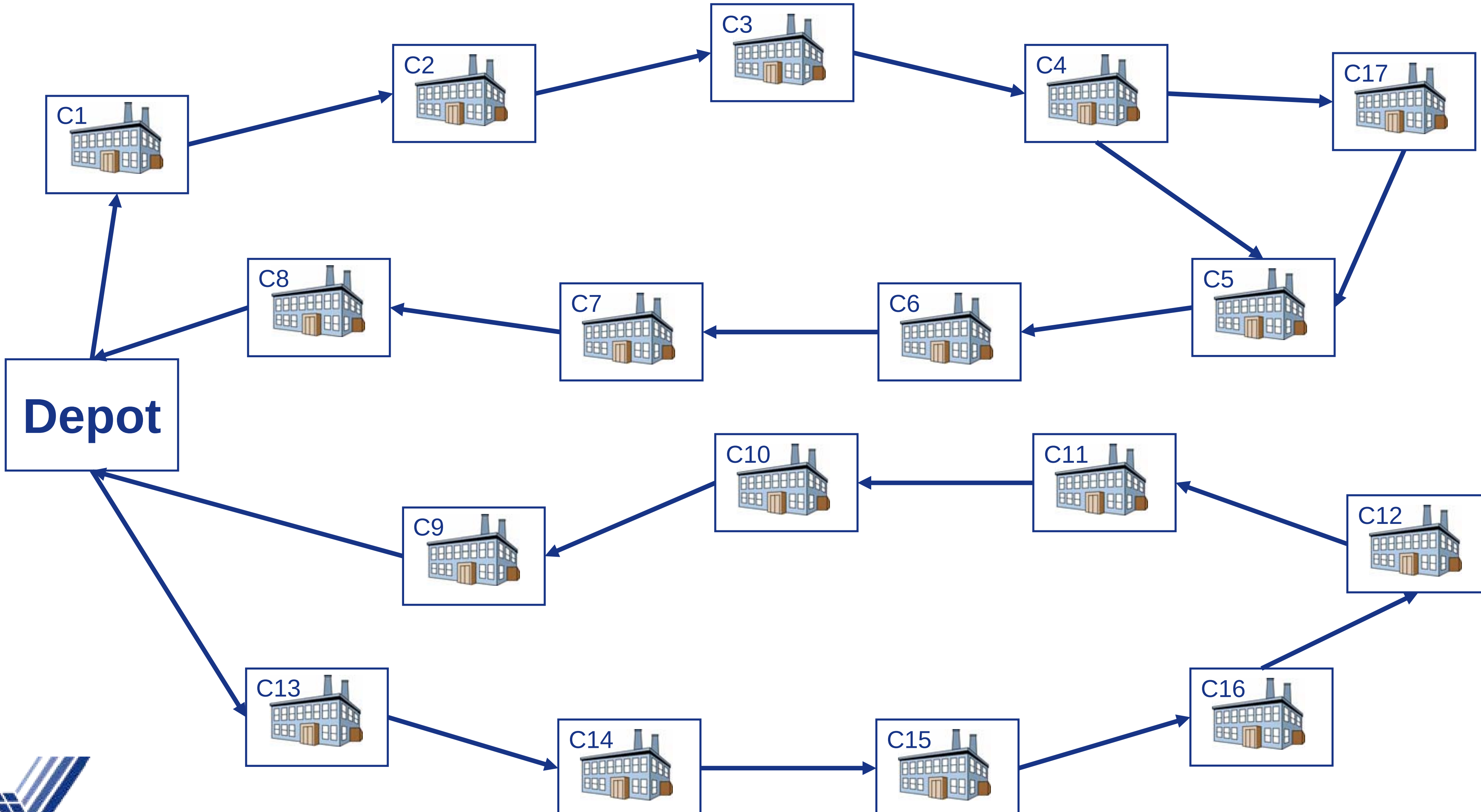
customers = n, # routes = m

- **Single Route Operators**
  - **InsertCustomer, RemoveCustomer: O(1)**
  - **CheapestInsertCustomer: O(n)**
  - 2 – OPT: $O(n^2)$
- Multiple Route Operators
  - Single Customer Operators
    - Move: $O(n^2)$
    - Exchange: $O(n^2)$
    - Combined Move/Exchange: $O(n^2)$
  - Path Operators (Multiple adjacent customers, solution parts)
    - Concatenate Tour Pair: $O(m^2)$
    - Split Tour: O(n)
    - Path Exchange: $O(n^4)$
    - Restricted Path Exchange (one end fixed to be a depot): $O(n^2)$

Operator: Insert Customer



Operator: Remove Customer

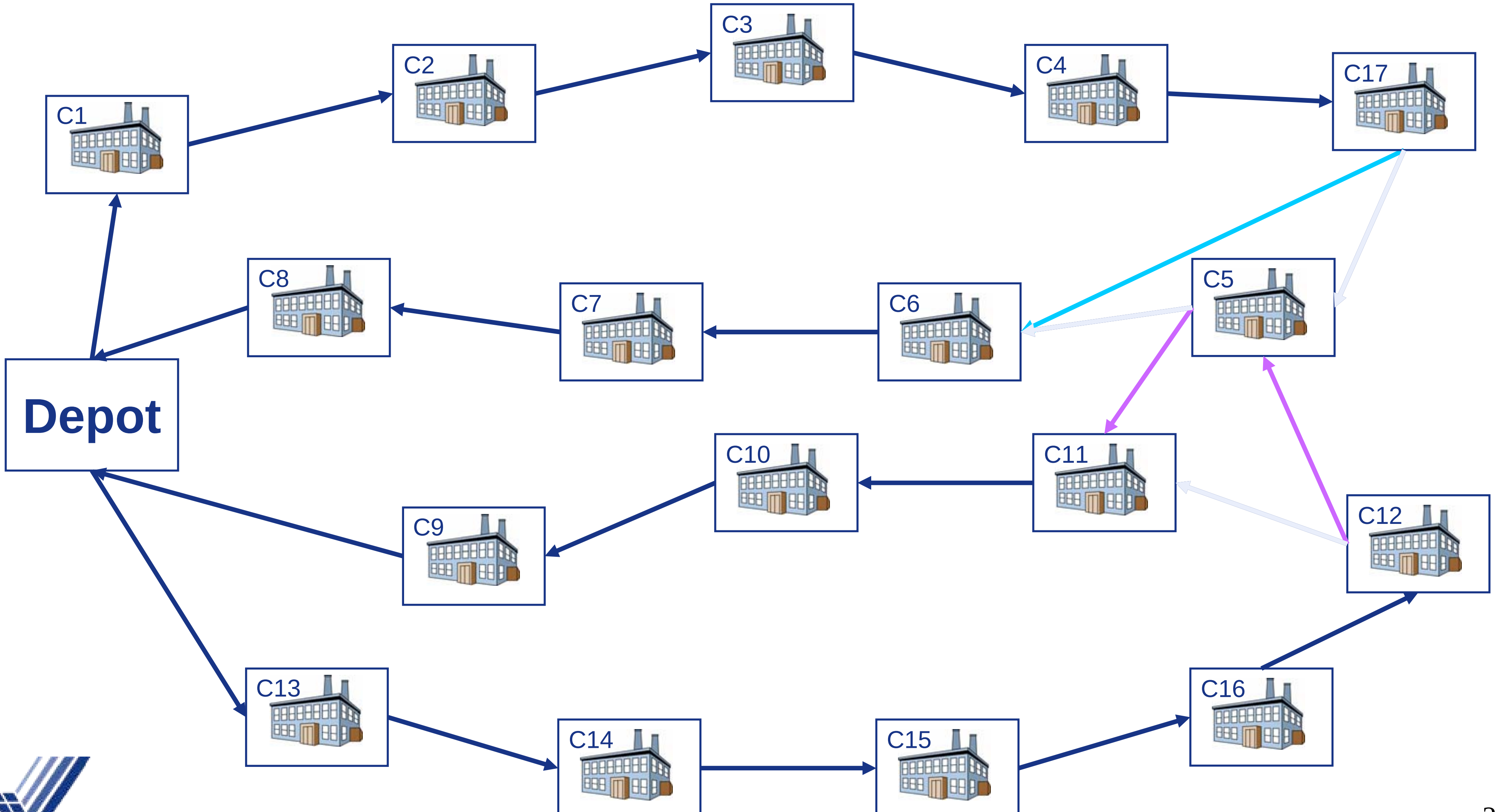


Operators / Neighborhoods / Neighborhood Size

customers = n, # routes = m

- Single Route Operators
  - InsertCustomer, RemoveCustomer: $O(1)$
  - CheapestInsertCustomer: $O(n)$
  - 2 – OPT: $O(n^2)$
- **Multiple Route Operators**
  - **Single Customer Operators**
    - **Move: $O(n^2)$**
    - **Exchange: $O(n^2)$**
    - **Combined Move/Exchange: $O(n^2)$**
  - Path Operators (Multiple adjacent customers, solution parts)
    - Concatenate Tour Pair: $O(m^2)$
    - Split Tour: $O(n)$
    - Path Exchange: $O(n^4)$
    - Restricted Path Exchange (one end fixed to be a depot): $O(n^2)$

Move Customer = Remove Customer + Insert Customer

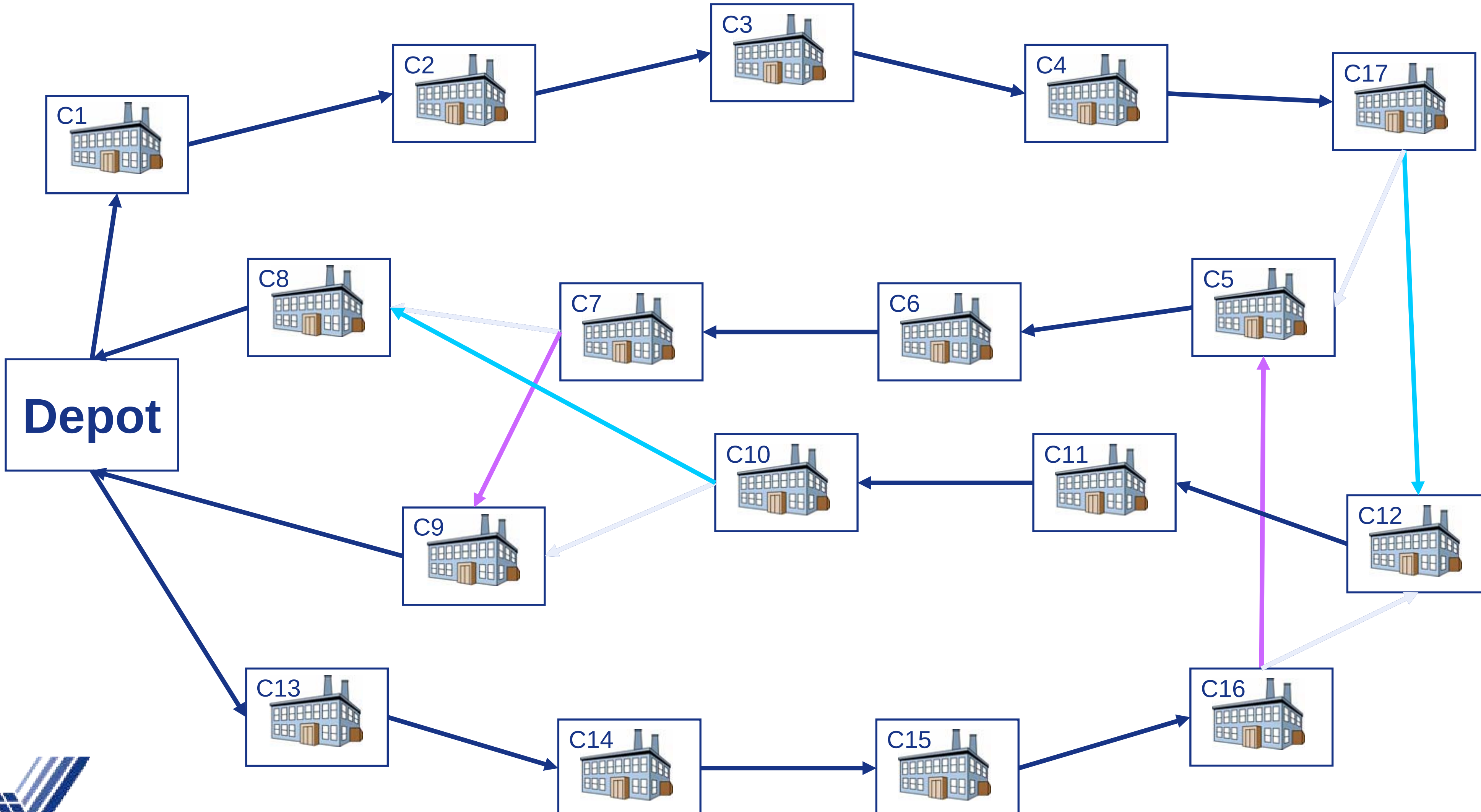


Operators / Neighborhoods / Neighborhood Size

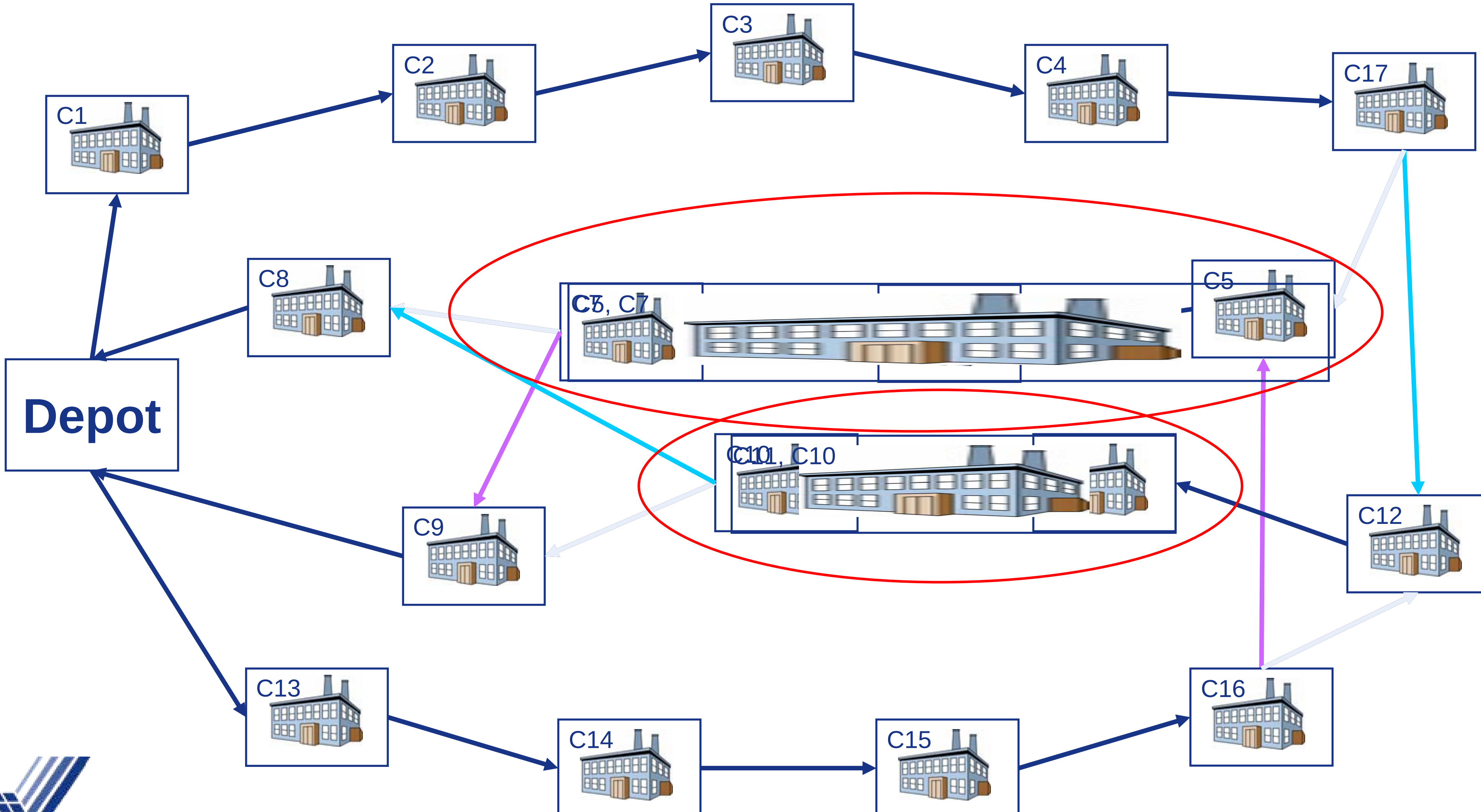
customers = n, # routes = m

- Single Route Operators
  - InsertCustomer, RemoveCustomer: $O(1)$
  - CheapestInsertCustomer: $O(n)$
  - 2 – OPT: $O(n^2)$
- **Multiple Route Operators**
  - Single Customer Operators
    - Move: $O(n^2)$
    - Exchange: $O(n^2)$
    - Combined Move/Exchange: $O(n^2)$
  - **Path Operators (Multiple adjacent customers, solution parts)**
    - **Concatenate Tour Pair: $O(m^2)$**
    - **Split Tour: $O(n)$**
    - **Path Exchange: $O(n^4)$**
    - **Restricted Path Exchange (one end fixed to be a depot): $O(n^2)$**

Operator: ExchangePath



Operator: ExchangeSuperCustomer



Constraint Handling and Acceleration Techniques

- Super Customer Concept for Accelerated Delta Function Evaluations of Path based Neighborhood Generating Operators
- Super-Customer Matrix, Fast Super-Customer Lookup Object, Hash Tables
- Reusing information of already visited and overlapping Neighborhoods
- Priority Lists
- Static or dynamic Neighborhood reduction, Candidate or Tabu Lists
- Efficient Data Structures

Partial Fixing of Decision Variables

- “Neighborhood Specific Local Optima Flags” for parts of the solution:
  - Customers (or subsets of customers)
  - Routes (or subsets of routes)
  - Routes assigned to a depot (or a subset of depots)
  - Routes assigned to a day (or a sub period)
  - Partial solutions according to a decomposition scheme

Decomposition in Sub Problems and Large Neighborhoods

- Hierarchical decomposition
  - PMDVRP, PVRP, PTSP
  - MDVRP, MDTSP
  - CVRP
  - TSP

- Select a series of different subsets of Routes
  - Geographical decomposition
  - Disjoint (Parallelization)
  - Overlapping
  - VNS-Scheme: Increasing number of routes

Scheme of a Hybrid (1+1)-Evolutionary Strategy

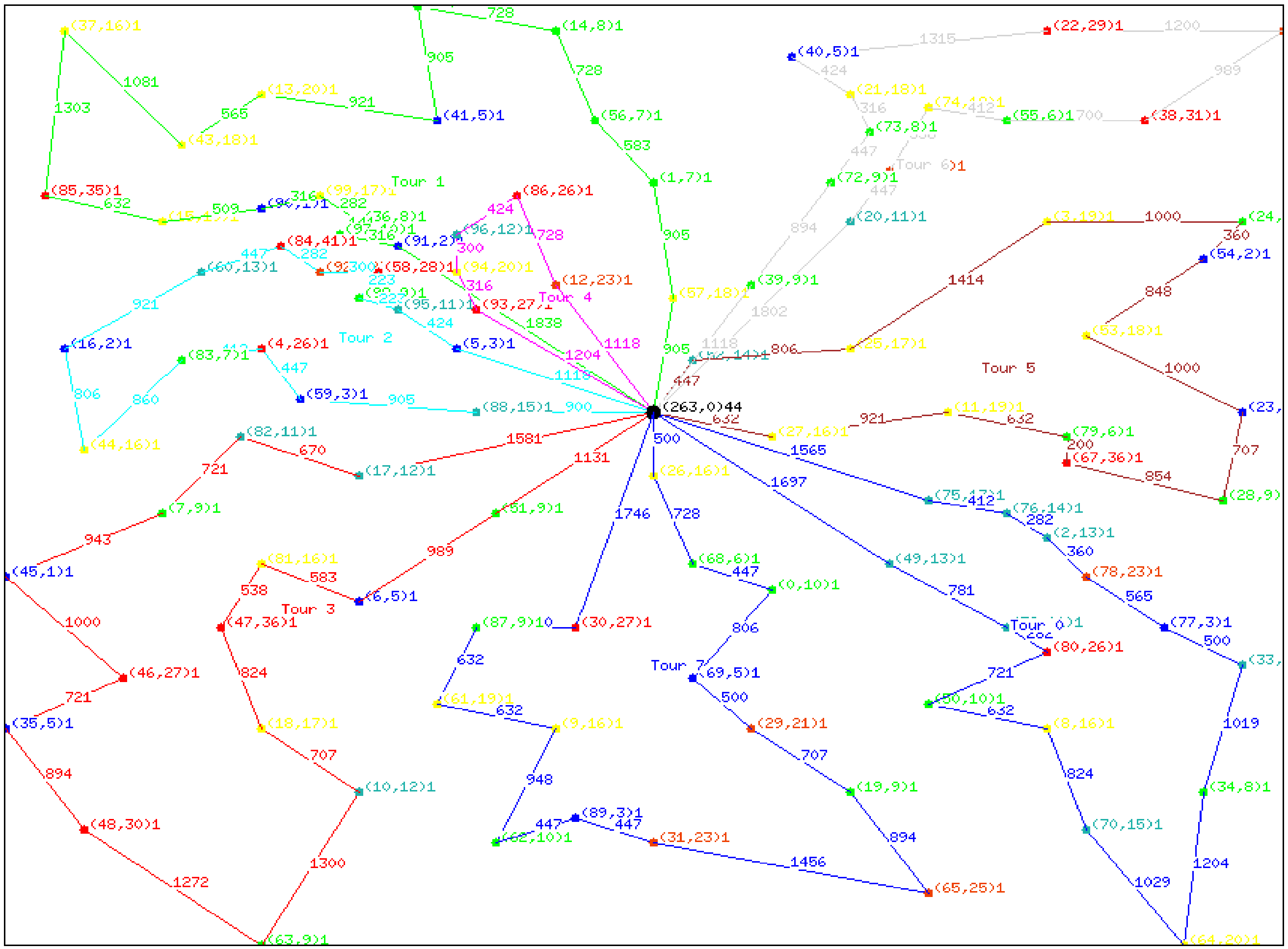
```
GNr := 0;
LastImprovingGeneration:= 0;
Initialization( Parent );
VariableNeighborhoodSearch ( Parent );
REPEAT
        GNr := GNr+1;
        Child := Mutation( Parent );
        VariableNeighborhoodSearch ( Child );
        if ( Fitness( Child ) => Fitness( Parent ) )
                Parent := Child;
                LastImprovingGeneration:= GNr;
UNTIL StoppingCriteria( GNr, LastImprovingGeneration, FitnessThreshhold );
```

Monday

CostNorm 825.76 TourLEu 826.14 Cost 82576 Length: 82576 VU: 1458

Nr	0	1	2	3	4	5	6	7
TourL	11873	12658	8268	13874	4090	9821	10602	11390
TourLC	11873	12658	8268	13874	4090	9821	10602	11390
Costs	11873	12658	8268	13874	4090	9821	10602	11390
VU	199	198	196	199	108	165	194	199
TourLEu	118.79	126.66	82.73	138.79	40.91	98.25	106.06	113.93

SAVE Tour | RESTORE Tour | Close! | Delay | DrawVU | DrawAngle | DrawTour



Competition with other methods

- **Quality of the best solutions found**
- **Solution quality versus computation time**

Hj.	Hjoring	"GRASP", "Tabu Search", "Genetic Algorithm"
GHL	Gendreau, Hertz, Laporte	"Tabu Search"
Osman	Osman	"Simulated Annealing", "Tabu Search"
Wark	Wark	"Repeated Matching Heuristic"
XK	Xu, Kelly	"Network Flow-Based Tabu Search"
Taillard	Taillard, Rochat	"Tabu Search", "Adaptive Memory Programming"
CB	Christofides, Beasley	"Initialization and Improvement Heuristic"
Paletta	Paletta	"PTSP - Heuristic"
CGW	Chao, Golden, Wasil	"Initialization and Improvement Heuristic"
CGL	Cordeau, Gendreau, Laporte	"Tabu Search"
TB	Tan, Beasley	"Generalized Assignment Heuristic"
RG	Russel, Gribbin	"Multiphase Approach"
Prins	Prins	"Evolutionary Algorithm"
MB	Mester, Bräysy	"Hybrid Evolutionary Strategies"
LC	Le Bouthillier, Crainic	"Parallel Cooperative Search"
Ropke	Ropke et. al.	"Adaptive Large Neighborhood Search"
Reinholz	Reinholz	"Hybrid (1+1) – Evolutionary Strategy"

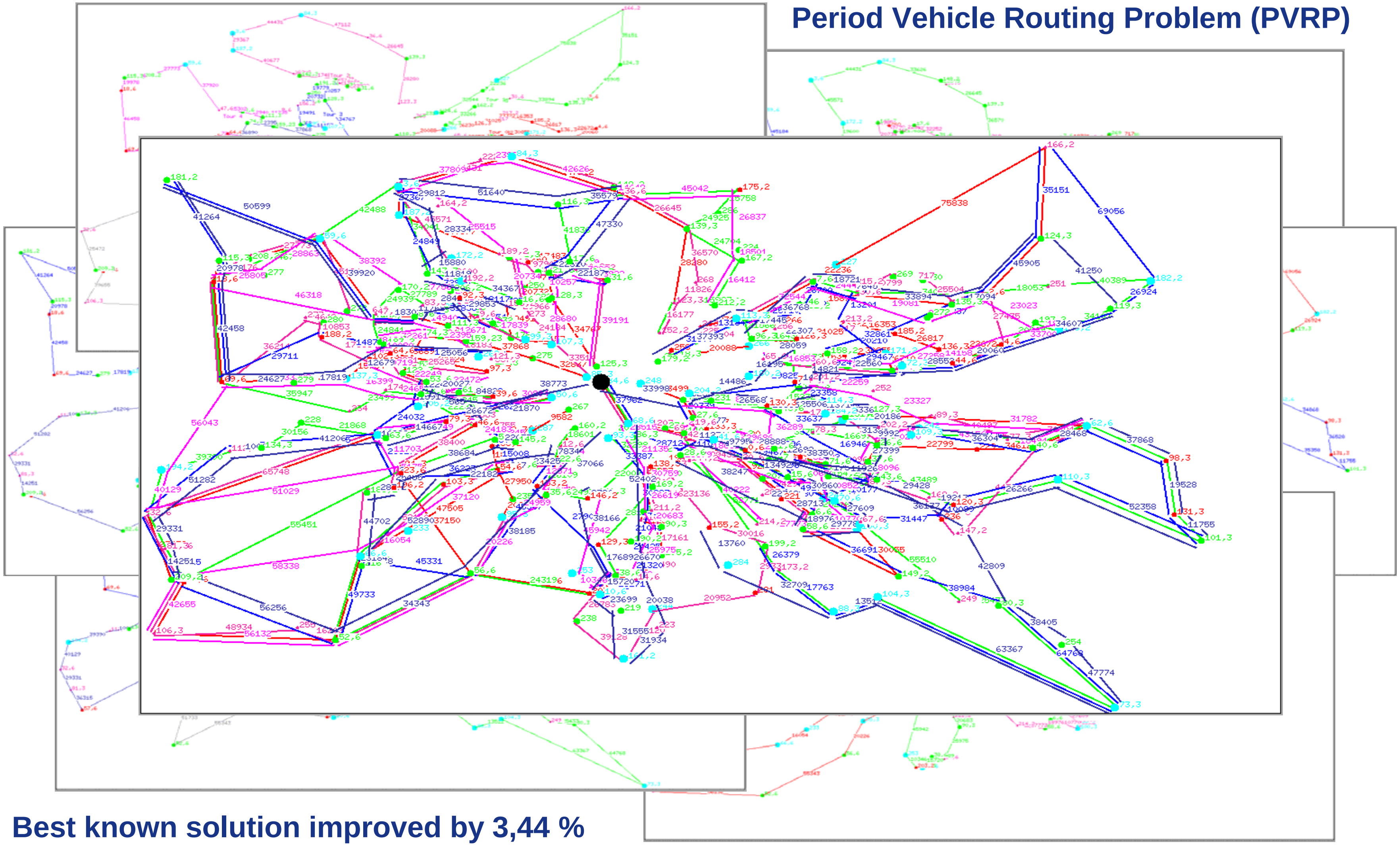
Benchmarks

CVRP	41 Instances,	41 best known solutions,	12 improved,	39 still best, 1995/1996
PTSP	33 Instances,	33 best known solutions,	15 improved,	33 still best, 1996/1997
PVRP	42 Instances,	42 best known solutions,	31 improved,	42 still best, 1996/1997
MDVRP	33 Instances,	33 best known solutions,	14 improved,	25 still best, 1997/2008
LSVRP	30 Instances,	33 best known solutions,	30 improved,	30 still best, 2001
OVRP	10 Instances,	10 best known solutions,	6 improved,	10 still best, 2008

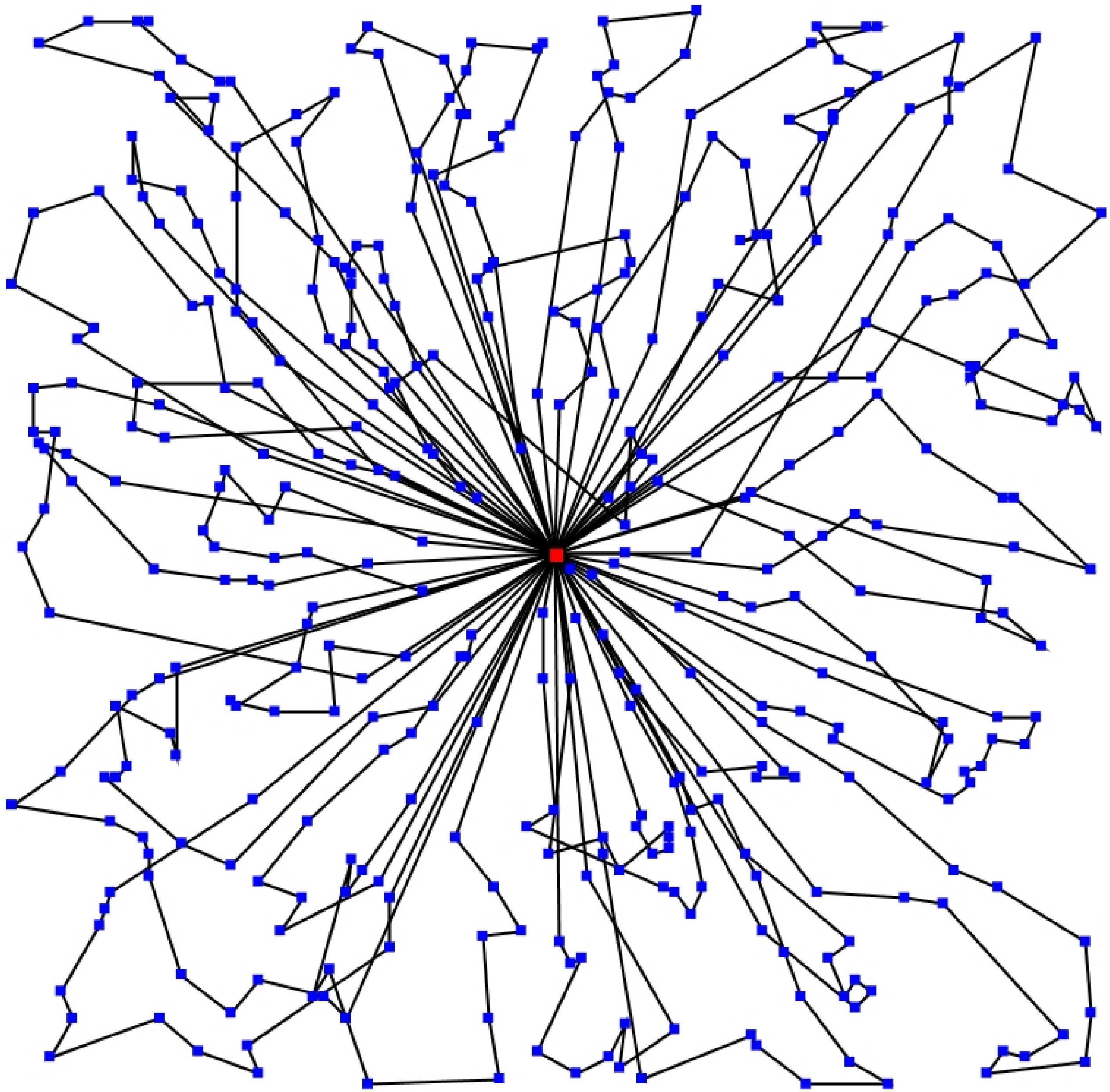
SUM 193 Instances, 189 best known solutions, 112 improved, 178 still best

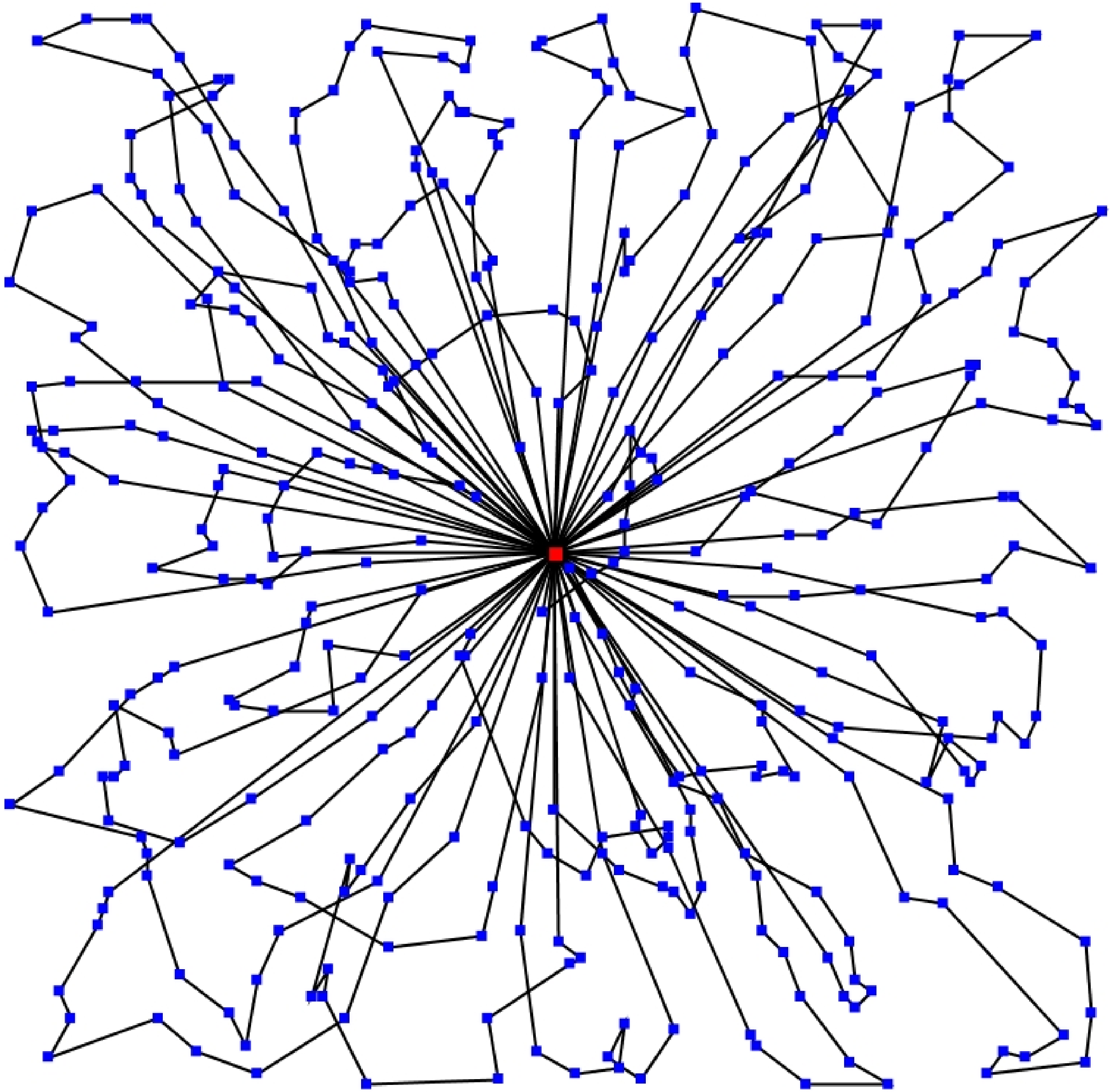
Robustness: The same Parameter Settings for all Instances of a Problem

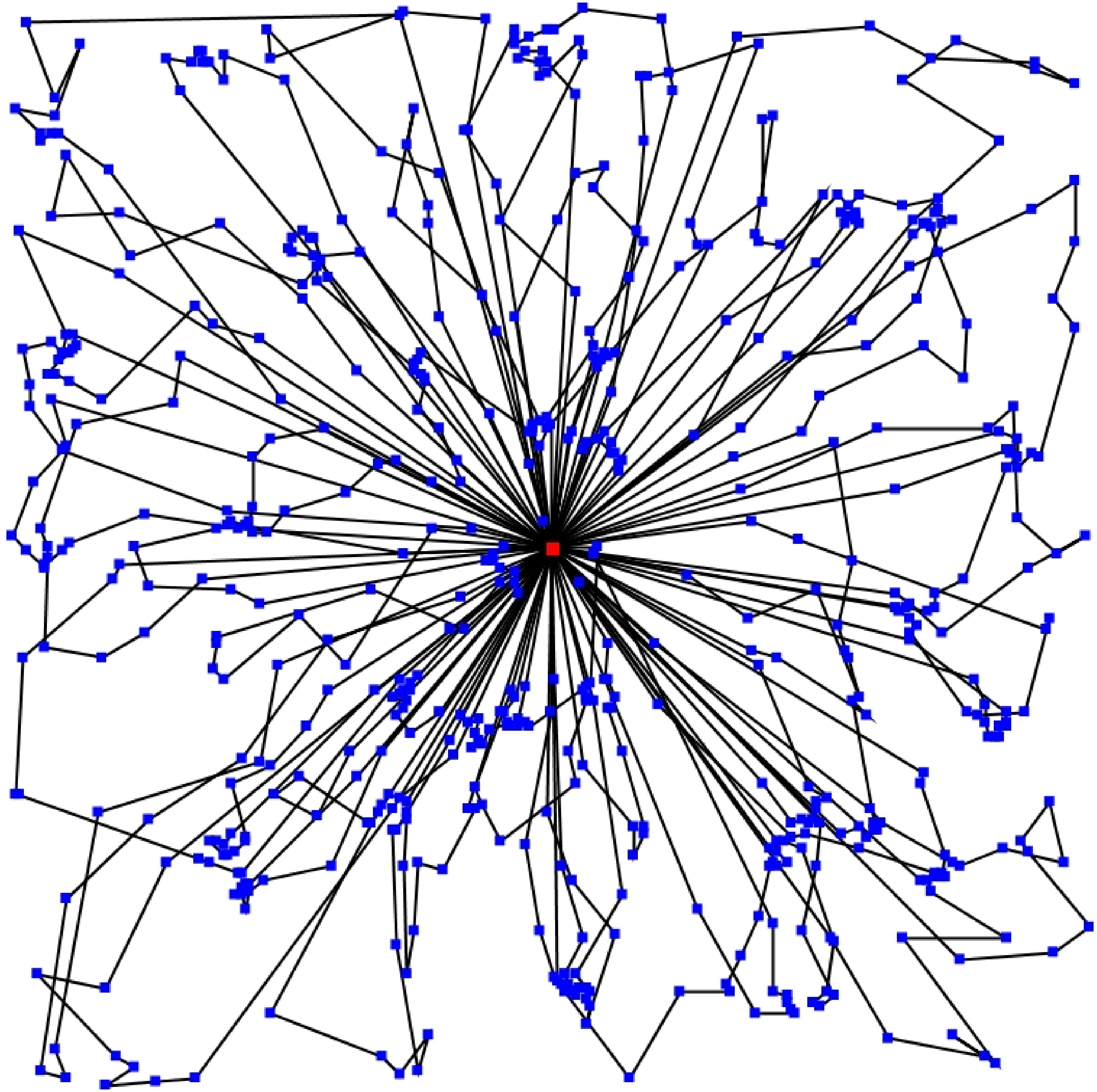
Period Vehicle Routing Problem (PVRP)



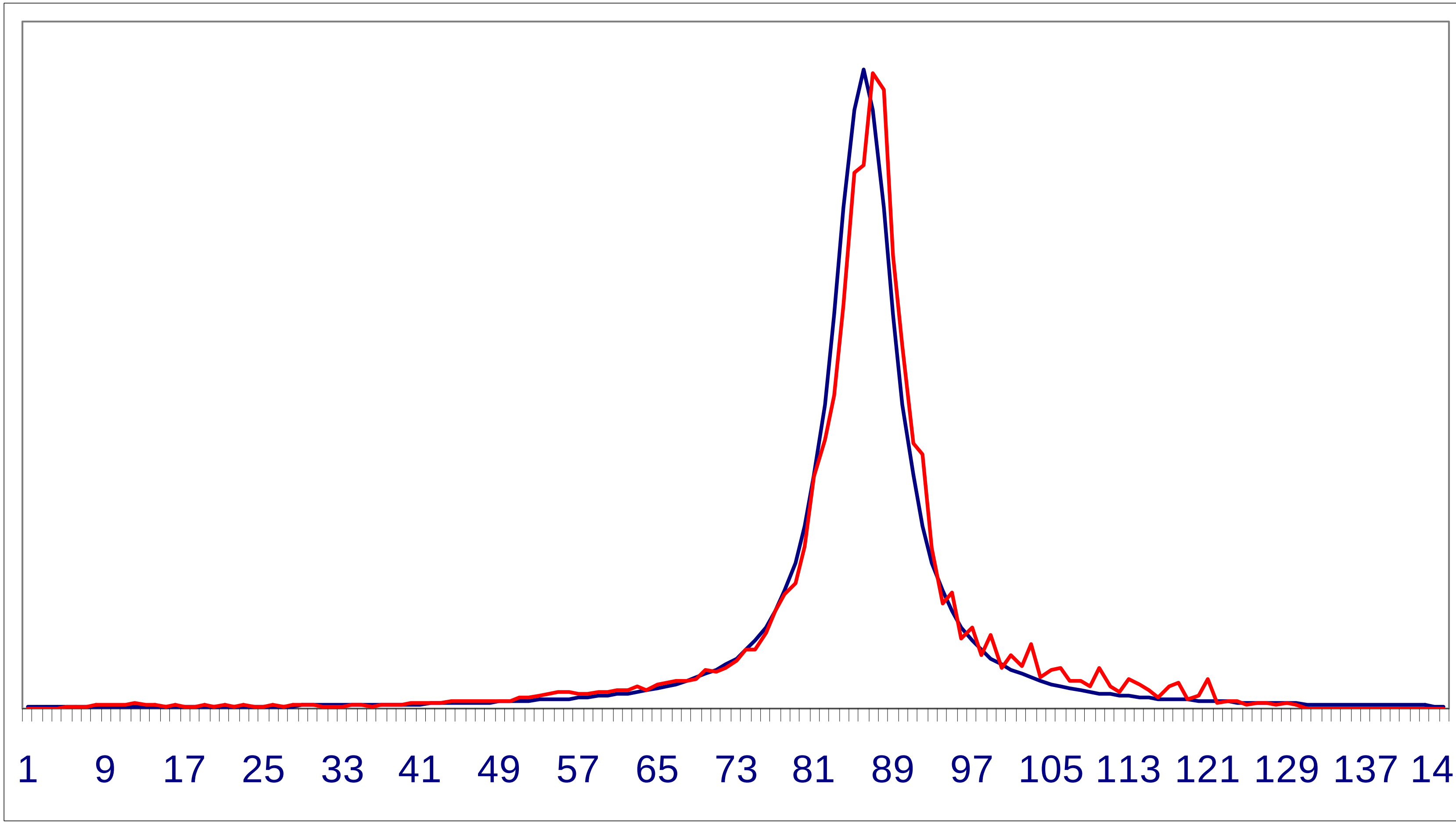
Best known solution improved by 3,44 %





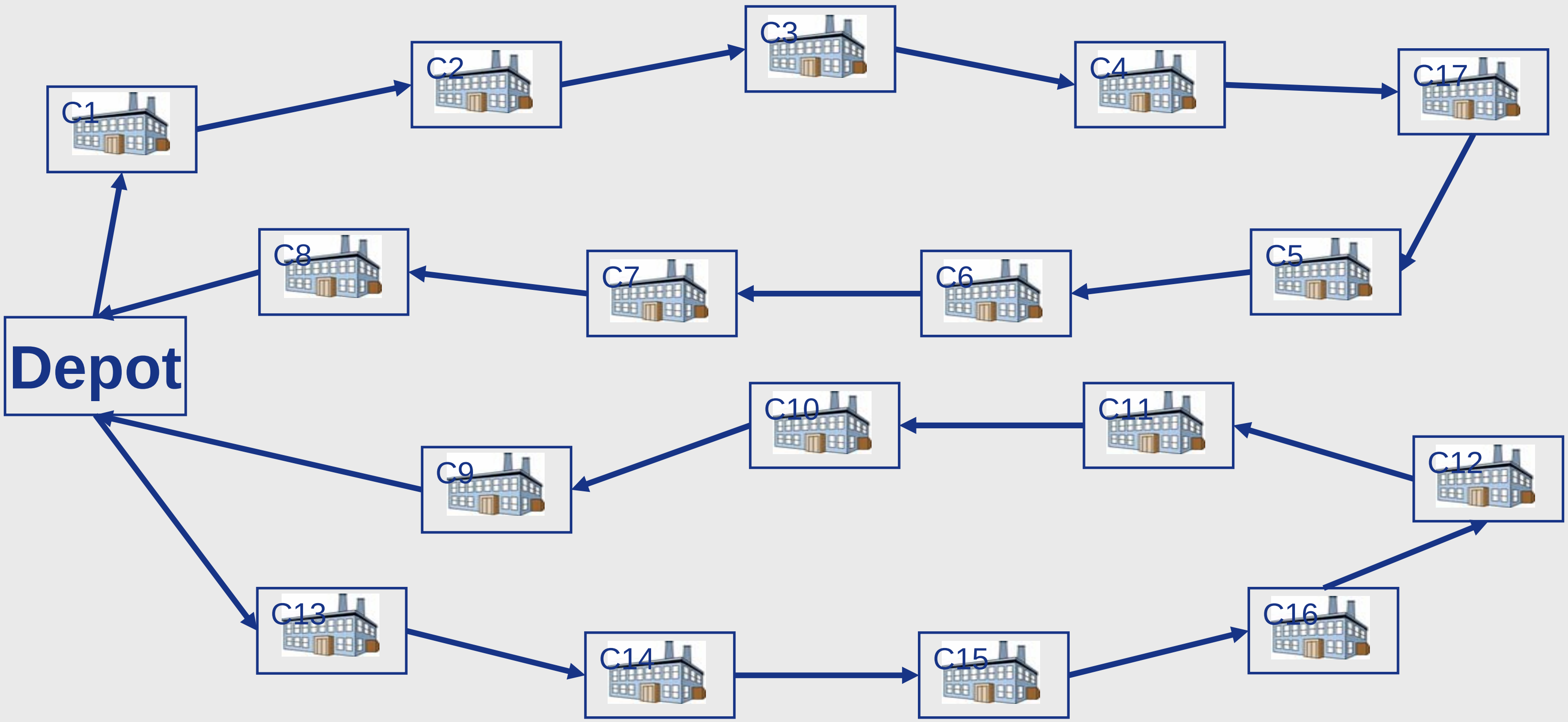


Uncertainty: Fitting Speed with a Cauchy Distribution

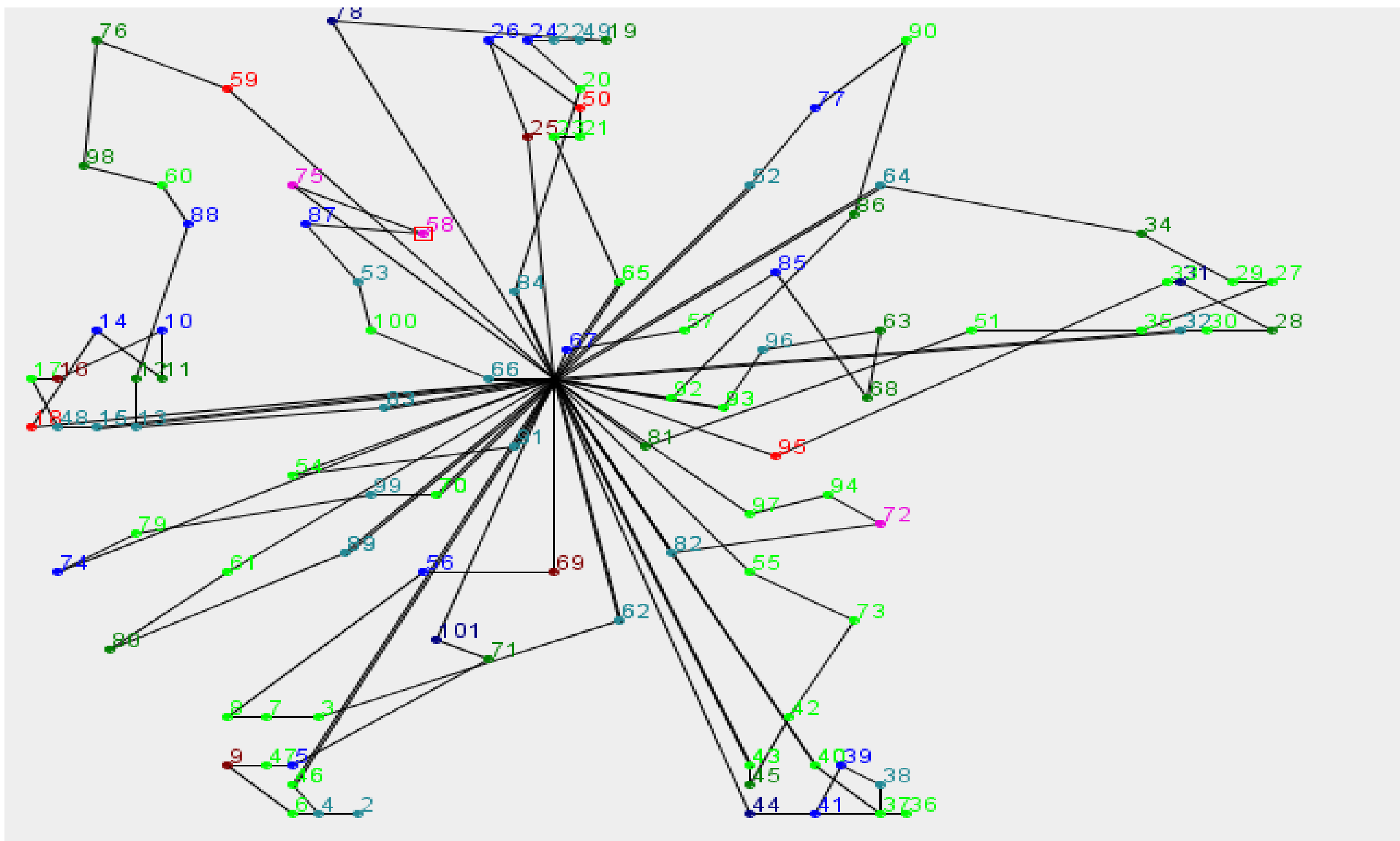


Modeling Uncertainty and Risk with Resources

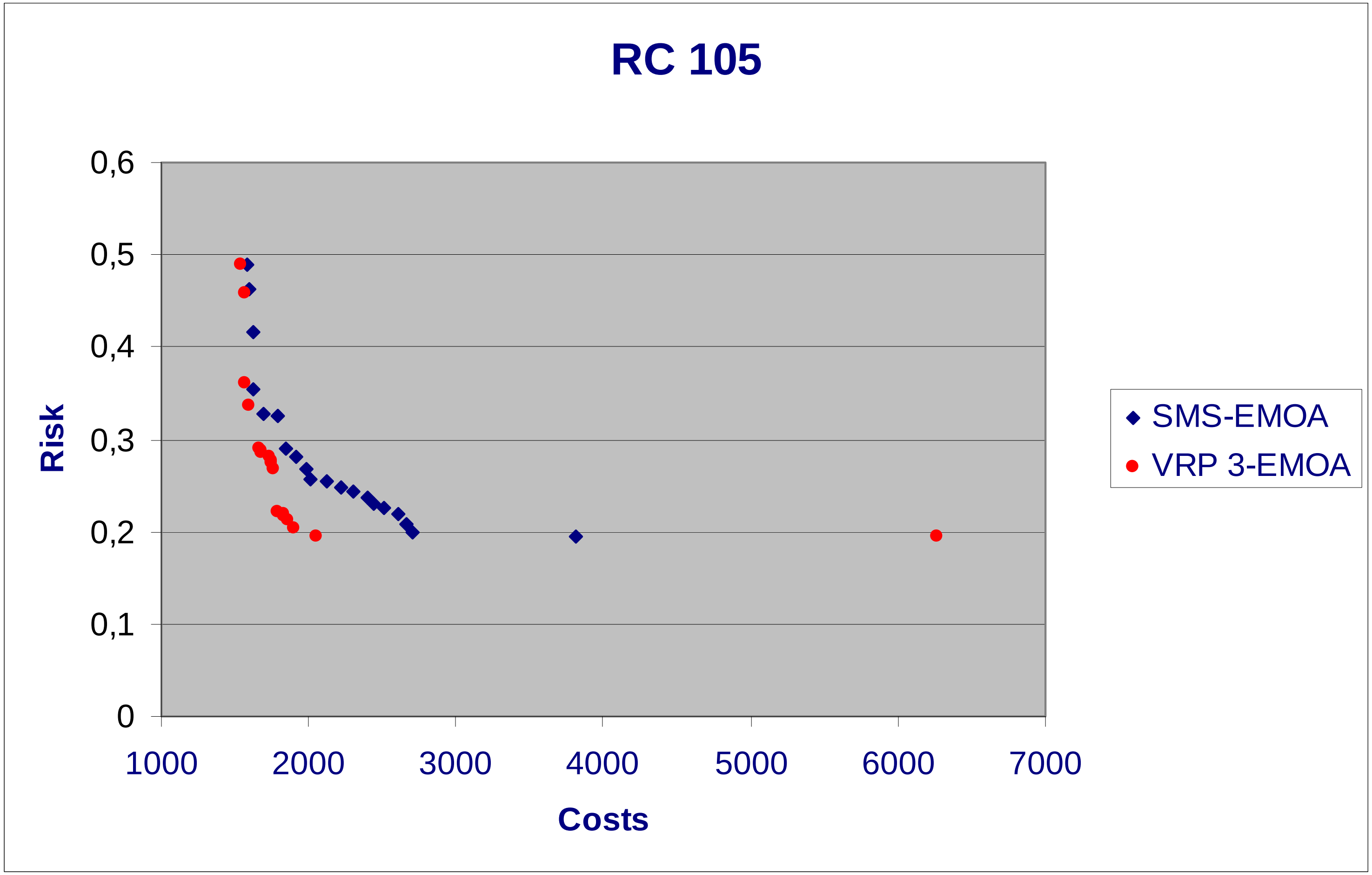
- Probabilistic Resource Consumption
- Series of Conditional Probability Distributions



Risk Management and Visualization



Tradeoff between Costs and Risk



Real World Rich Vehicle Routing Problem

- Single Unit Multiple Depot VRPTW with Backhauls
- Heterogeneous fleet
- Capacity limits
- Multiple compartments with dynamic adjustable compartment sizes
- Load restrictions, LIFO, Inner compartment dependencies,
- Fixed costs
- Customer dependent costs
- Asymmetric distance and estimated driving times on a real road network
- Tour length limits, operation time limits
- Customer specific service times, back on road network times
- Traffic flow factor
- Flexible starting times
- Multiple Depots
- Cross Docking

Ongoing Research and Outlook

- More constraints
- Noise, incomplete and uncertain data, robustness, risk management
- Dynamic and Online Optimization
- Multi-criteria Optimization
- Enhanced variation mechanisms
- Adaptive or self-adaptive disruption strategies
- Adaptive strategy control mechanisms (Species, Agents)
- More complex and hierarchical nested optimization problems
- More Real World applications